

Optimal Fiscal and Monetary Policy with Nominal and Indexed Debt

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Abstract

This paper examines the importance of debt composition in the setting of optimal fiscal and monetary policy over the business cycle. The main conclusion is that fixed-rate nominal debt as state-contingent debt can be a significant policy tool to reduce volatility of distortionary government policy and thereby reduce macroeconomic volatility. The gain in welfare from using nominal debt to hedge against shocks to the government budget is equal to 0.7 percent of lifetime utility for the United States. Employing nominal debt for hedging purposes has limits however and, based on the outstanding level of debt, the current U.S. debt composition of 95 percent nominal debt and 5 percent inflation-indexed debt appears optimal.

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1. Introduction

A fundamental issue facing policymakers is the role of government debt in the setting of optimal fiscal and monetary policy.¹ The subject of debt composition is of particular importance since a variety of return structures exist in debt markets. A canvass of sovereign debt markets indicates that some governments have debt structures with predominantly fixed-rate nominal coupon debt while others primarily issue debt with returns indexed to the exchange rate, interest rate, or general price index.² This paper examines the role of sovereign debt composition in the setting of optimal fiscal and monetary policy over the business cycle and its impact on the ability of debt to serve as a shock absorber.

Many economists like Bach and Musgrave (1941), Lucas and Stokey (1983), and McCallum (1984) have advocated the use of inflation-indexed debt, or fixed-rate real debt, as a means to eliminate the incentive for governments to inflate away existing nominal liabilities. However, more recently Goldstein (2002) and Reinhart, Rogoff, and Savastano (2003) have argued that an indexed debt structure can be destabilizing if large shocks are transmitted into additional debt costs at inopportune times, as seen recently in several emerging market crises. As an alternative to indexation Bohn (1988) and Chari et al. (1991) suggest fixed-rate nominal debt could hedge against unforeseen economic shocks that impact the government budget. However, neither tests this idea in a general equilibrium setting. This paper takes advantage of recent advances in computing technology and numerical approximation procedures in order to calibrate and simulate a stochastic monetary model under various debt-to-income ratios and differing debt compositions. The government in this economy has the ability to issue two types of debt: one-period nominal debt and one-period real debt. Since this analysis assumes a closed economy, all real debt is modeled as inflation-indexed debt.

The main conclusion is that the role of fixed-rate nominal coupon debt as state-contingent debt can be a significant policy tool to reduce volatility of distortionary fiscal and monetary policy. Economies with nominal debt are shown to be less volatile than economies with real debt since nominal debt acts as a hedge against unexpected shocks to the government budget. Unexpected shocks to the economy that call for an increase in distortionary govern-

¹The tax smoothing models of Barro (1979, 1987), which parallel Friedman's permanent income hypothesis, view debt as an additional policy tool that allows for less volatile distortionary taxes over time. Other authors have focused on the intergenerational aspects of debt. Musgrave (1959) for example suggested that the financing of government expenditures should be based on the principle of who benefits from such spending. According to this principle current spending should be financed from current revenues while capital spending, including wars, should be financed over time since the benefits accrue across generations. Finally, a more recent line of research combines the public finance tradition (Ramsey 1927) with general equilibrium macroeconomics to examine the setting of monetary and fiscal policy over the business cycle. Lucas and Stokey (1983) and Chari, Christiano, and Kehoe (1991, Hereafter referred to as Chari et al.) examine how taxes on debt or partial defaults can be used to make the return on debt state-contingent.

²At the end of 2002, approximately 95 percent of U.S. federal debt outstanding was in the form of fixed-rate nominal coupon debt while 5 percent was in the form of inflation-indexed debt, the U.S. Treasury Inflation-Indexed Securities. In contrast, Brazil's domestic public debt profile was almost exclusively composed of debt with returns indexed to domestic interest rates (46 percent), the exchange rate (38 percent), and domestic inflation (13 percent). Only 3 percent was in fixed-rate nominal coupon debt. Figures are based solely on domestic currency denominated government debt. External debt denominated in foreign currency is not included. The exchange rate-linked domestic debt includes the issuance of bonds and swaps.

ment revenue also correspond to states with higher-than-expected inflation which reduces the value of pre-existing nominal debt and works against the need to increase distortionary labor taxes or money growth. In states of the world with positive shocks to government spending and negative shocks to technology, the shadow value of reducing debt is increased, creating a positive correlation between inflation and the multiplier on the government budget constraint. In this setting nominal debt acts as a hedge against shocks to the government budget by providing a non-distortionary source of revenue, permitting a smoother path for fiscal and monetary policy and, in turn, household consumption. Depending on the type and magnitude of economic shocks that prevail, the results in this paper suggest that optimal debt policy should include nominal debt in sufficient quantities to reduce macroeconomic volatility and minimize costs associated with the business cycle.

The calculated welfare gains from the hedging role of nominal debt are significant in that they are roughly equivalent in magnitude to the gains from reducing business cycle volatility or eliminating the costs of moderate inflation as suggested in other studies. Employing the optimal percentage of nominal debt for hedging purposes results in welfare gains similar in magnitude to those reported by Lucas (1987), who estimates the cost of business cycles using pre- and post-World War II consumption data, and by Cooley and Hansen (1991), who estimate the gains from eliminating moderate inflation. Simulation results also indicate the gain in welfare from using nominal debt as opposed to real debt is roughly similar in magnitude to the gain in welfare from the ability to issue real debt alone (Cosimano and Gapen 2004).

While nominal debt can improve policy smoothing and household welfare, the hedging role of nominal debt has limits. Too much nominal debt eventually begins to increase the volatility of government policy as changes in the real value of the debt begin to outweigh the direct effects of shocks to the government budget. Given the existing level of debt in the U.S. economy at end-2002, simulations indicate that the existing split between nominal and real debt is optimal for policy smoothing purposes. If the U.S. were to accumulate additional debt resulting in increases in the debt-to-income ratio, this study suggests that this should be accomplished with higher proportions of real debt, in this case inflation-indexed debt. Under the high debt-to-income ratio, which is calibrated to twice the 2002 U.S. level of debt, model simulations indicate the optimal debt composition from a hedging perspective requires a blend of 65 percent nominal debt and 35 percent inflation-indexed debt.

The conclusions of this paper also highlight more generally the importance of time consistent government policy. Without a commitment mechanism in place, the government is unable to credibly commit to a future series of policy actions beforehand, leaving policy discretionary. As discussed in Lucas and Stokey (1983), the government will have an incentive to inflate away fixed-rate nominal liabilities unless all prices are predetermined or distortionary taxes can be avoided. If taxes are distortionary, the incentive to inflate away nominal liabilities is diminished, but not entirely removed. The lack of a commitment mechanism may be the factor that leads to highly indexed debt structures to begin with as issuers shoulder additional risks the market is unwilling to bear in order to place additional quantities of debt. Time consistent government policy therefore may be a necessary

precondition to enable full implementation of optimal debt policy.³ This may require governments to focus on commitment strategies sustained by reputational mechanisms as discussed in Chari, Kehoe, and Prescott (1989), Chari and Kehoe (1990, 1993), Stokey (1991), and Ljungqvist and Sargent (2000).

The characterization of the policies that generate the Ramsey equilibrium theoretically is difficult since the system is nonlinear. The presence of nonlinear distortions to labor requires the use of a simulation procedure which captures these effects. As discussed in Kim and Kim (2003), traditional log-linearization techniques may not produce accurate welfare computations since it neglects higher moments which are essential for measuring overall risk and, therefore, estimation of welfare gains. In order to capture these higher moments, this paper employs the projection method of Judd (1992, 1998) as applied to Ramsey problems in Cosimano and Gapen (2004) to solve the Euler conditions for optimal money growth, taxes, and debt. The Ramsey problem is solved in economies with various combinations of nominal and indexed debt. The low-debt economy uses the prevailing debt-to-income ratio in the United States and the high-debt economy is calibrated to twice this level and the composition of debt is varied between a full indexed debt policy to a full nominal debt policy. Then using the optimal Ramsey plan (policy choices of the government, allocations by the household, and the resulting price system), each economy was simulated under the effects of technology and government spending shocks in order to examine the affects of debt structure on optimal policies and activity.

The properties of the optimal endogenous solutions to fiscal and monetary policy are generally consistent with findings in the real business cycle literature, but with some differences. Regardless of the debt structure, optimal endogenous government policy follows the Friedman rule which results in an expected gross nominal interest rate equal to 1. In enacting this monetary policy rule, the government equates the real gross rate of return across the three assets (money balances, nominal debt, and indexed debt) in expectation, satisfying Euler conditions. According to Alvarez, Kehoe, and Neumeyer (2004) and Chari et al. (1991, 1996), the Friedman rule is optimal in a variety of monetary economies with distorting taxes and this paper extends this result to include a variety of different debt specifications. One main difference is that labor tax rates fluctuate to preserve stability of money growth rates which is the opposite result of Schmitt-Grohé and Uribe (2004) and Chari et al. (1991), yet volatility of labor taxes does not appear to be extreme and is consistent with available U.S. data. Within the business cycle, monetary policy is countercyclical with respect to technology shocks and procyclical with respect to government consumption, but only under economies with mostly indexed debt. The result is reversed for economies with mostly nominal debt. Finally, the economies with more nominal debt display liquidity effects through a negative correlation between money growth and real interest rates.

³See Alvarez et al. (2004) for analysis on how the terms of nominal and indexed government debt can be structured to support time consistent Ramsey policies in monetary economies. The authors also show under what conditions the optimality of the Friedman rule yields a time consistent policy and vice versa.

2. A stochastic monetary economy

The properties of debt composition and its relation to optimal policies and allocations are examined in a stochastic monetary economy. The model is a combination of a cash-in-advance model and a stochastic growth model, similar to models employed in Cooley and Hansen (1995), Chari et al. (1991), and Lucas and Stokey (1983). The economy is populated by a representative household, a representative firm, and a government. The household has logarithmic preferences and derives utility from leisure and two consumption goods, a cash good and a credit good, where previously accumulated cash balances are needed to purchase units of the cash good. Output is produced according to a production function that combines a constant capital stock with variable labor and technology, where the process governing technology is assumed to be exogenous and stochastic. The government raises revenue with distortionary effects to finance its exogenous stochastic spending through a tax on labor income, printing money, or debt issuance. Like Lucas and Stokey (1983), Alvarez et al. (2004) and others, this framework does not include a tax on capital and therefore avoids the well understood problems arising from capital taxation.⁴ In addition to the level of debt, the government also has the ability to choose between two types of debt: one-period nominal or one-period real debt. All real debt in this closed economy is modeled as inflation-indexed debt.

Assumptions of a fixed capital stock and logarithmic preferences enable computation of closed form equilibrium solutions for the private sector given a particular government policy. The Ramsey equilibrium solves for optimal fiscal and monetary policy given the equilibrium behavior of the private sector. This Ramsey equilibrium may be reduced to four conditions for money growth, taxes, the shadow price of debt, and labor given the equilibrium behavior of interest and prices. The system is nonlinear and therefore the projection method is applied to solve for the four policy functions and conduct simulations. If the private sector is made more complex, these four conditions would need to be augmented with equilibrium conditions for interest rates and prices. These additional conditions would limit the accuracy of the projection method since additional equations would limit the number of nodes the computer can solve. Finally, given a fixed capital stock, the model highlights the distortionary effects of labor taxes and, thus, the optimal government policy will account for its impact on interest rates and prices as well as the optimal behavior of the household and firms.

2.1. Production

Aggregate output, Y_t , is produced according to the following constant returns-to-scale production function,

$$Y_t = \exp(\theta_t) H_t^\alpha K_t^{1-\alpha}, \quad 0 < \alpha < 1, \quad (2.1)$$

where K_t and H_t are the aggregate capital stock and labor supply, respectively, and θ_t represents the available technology. Technology is assumed to be the realization of an

⁴In addition to ruling out taxation of the pre-existing stock of capital, an assumed zero capital tax is also justified by the well established result that tax rates on capital should be close to zero on average. For other work on optimal capital income taxes, see Atkinson (1971), Diamond (1973), Pestieu (1974), Atkinson and Sandmo (1980), Judd (1985), Chamley (1986), and Chari et al. (1991, 1994).

exogenous stochastic process and evolves according to the following law of motion,

$$\theta_t = \rho_\theta \theta_{t-1} + \epsilon_{\theta,t}, \quad 0 < \rho_\theta < 1. \quad (2.2)$$

The random variable, $\epsilon_{\theta,t}$, is normally distributed with mean zero and standard deviation $\sigma_{\theta,t}$ and the realization of $\epsilon_{\theta,t}$ is known to all agents at the beginning of period t .

The production function in (2.1) has meaningful implications which differ from similar recent work by Aiyagari et al. (2002), Alvarez et al. (2004), and Schmitt-Grohé and Uribe (2004). These authors all assume $\alpha = 1$ which results in exogenous marginal product of labor. In contrast, the restriction in this paper on labor's share of income below unity means labor supply is nonlinear and marginal product of labor is endogenous.⁵ As discussed in the proceeding sections, the solution procedure used in this analysis preserves the nonlinearity of the labor supply function and associated Jensen's inequality effects. Thus, preserving the endogeneity of the marginal product of labor captures the cost of government policy in a more comprehensive manner.

The portion of output that is not consumed is invested into physical capital. Investment in period t produces capital in period $t + 1$,

$$K_{t+1} = (1 - \delta) K_t + X_t, \quad 0 < \delta < 1, \quad (2.3)$$

where X_t is the level of investment in period t and δ is the rate of depreciation.

The assumption of a fixed capital stock implies $X_t = X = \delta K$ and firms are assumed to take depreciation charges before taxes are applied at the household level. If firms were not allowed to take depreciation charges before taxes were applied, the government would find it optimal to tax inelastically supplied investment and use the proceeds to retire money balances. The representative firm seeks to maximize profit by choosing labor supply resulting in the standard first-order conditions for the wage rate and rental rate on capital, adjusted for constant capital. A fixed capital stock implies investment is equal to depreciation and therefore income from the production process returned to the household each period is closely related to the concept of net national product.

2.2. Households

The representative household obtains utility from consumption and leisure. Preferences are summarized by the following utility function,

$$E_t \sum_{t=0}^{\infty} \beta^t [a \ln C_{1t} + (1 - a) \ln C_{2t} - \gamma H_t], \quad (2.4)$$

⁵For example, setting $\alpha = 1$ in (2.1) results in marginal product of labor equal to $\partial Y / \partial H = \exp(\theta)$. Setting $0 < \alpha < 1$ results in a marginal product of labor of $\partial Y / \partial H = f(\alpha, \exp(\theta), H, K)$.

where C_1 is a cash good, C_2 is a credit good, γ is a positive constant and $0 < \beta, a < 1$.⁶ The household enters period t with previously accumulated assets equal to the stock of money holdings, M_t , and gross returns from nominal government bonds, $B_t^N R_{t-1}^N$, and indexed government bonds, $B_t^L R_{t-1}^L$ (the superscript, L , is used to indicate that returns are inflation-linked). Here B_t^N and B_t^L represent the stocks of nominal and indexed debt, respectively, issued the previous period and R_{t-1}^N and R_{t-1}^L defines the gross nominal and real interest rates, respectively. These assets augment the income received from capital and the after-tax income from labor and are used to finance household expenditures during the period. Entering the period, the current shocks to technology and government spending are revealed. As a result of this specification, households know the past and current realization of technology and government spending and form expectations over future possible values. The household then chooses labor supply, consumption of the cash and credit goods, demand for nominal money balances to be carried into the next period, M_{t+1}^d , and stocks of nominal and indexed government debt. Overall, household allocations must satisfy the following budget constraint,

$$C_{1t} + C_{2t} + \frac{M_{t+1}^d}{P_t} + \frac{B_{t+1}^N}{P_t} + B_{t+1}^L \leq (1 - \alpha\tau_t)(Y_t - X) + \frac{M_t}{P_t} + \frac{B_t^N}{P_t} R_{t-1}^N + B_t^L R_{t-1}^L, \quad (2.5)$$

where P_t equals the price level and τ_t is the tax applied to labor income. Previously accumulated money balances are used to purchase the cash good in the current period and must also satisfy the cash-in-advance constraint,

$$P_t C_{1t} \leq M_t. \quad (2.6)$$

Output can either be consumed by households or used by the government resulting in the economy-wide resource constraint,

$$C_{1t} + C_{2t} + X + G_t = Y_t, \quad (2.7)$$

where G_t represents the level of real government consumption.

The specification of log preferences and a fixed capital stock allows for the derivation of closed-form solutions for consumption, prices, and interest rates given a particular set of government policy since the income and substitution effects cancel.⁷ The closed-form

⁶Consumption of the cash and credit good are derived from the same production function and are therefore perfect substitutes in production along with government spending. The specification that utility is linear in labor is discussed in Hansen (1985) and Rogerson (1988) and is based on the assumptions that labor is indivisible and allocation of labor is determined by employment lotteries.

⁷See Appendix for the solution to the household problem.

solutions for consumption and the price level are,

$$C_{1t} = \frac{(Y_t - X - G_t) \beta \left(\frac{a}{1-a}\right) \exp(-\mu_{t+1})}{1 + \beta \left(\frac{a}{1-a}\right) \exp(-\mu_{t+1})}, \quad (2.8)$$

$$C_{2t} = \frac{(Y_t - X - G_t)}{1 + \beta \left(\frac{a}{1-a}\right) \exp(-\mu_{t+1})}, \quad (2.9)$$

$$P_t = \frac{M_t}{(Y_t - X - G_t)} \left[\frac{1 + \beta \left(\frac{a}{1-a}\right) \exp(-\mu_{t+1})}{\beta \left(\frac{a}{1-a}\right) \exp(-\mu_{t+1})} \right], \quad (2.10)$$

where $\exp(\mu_{t+1}) = M_{t+1}/M_t$ defines the gross growth rate of money. The closed-form solutions for the real and nominal interest rate are found by inserting (2.9) and (2.10) at time t and $t + 1$ into

$$R_t^L = \frac{1}{\beta C_{2t}} \left[\frac{1}{E_t \left[\frac{1}{C_{2t+1}} \right]} \right], \quad (2.11)$$

$$R_t^N = \frac{1}{\beta C_{2t}} \left[\frac{1}{E_t \left[\frac{1}{C_{2t+1}} \frac{P_t}{P_{t+1}} \right]} \right]. \quad (2.12)$$

which are derived from the Euler conditions on nominal and indexed bonds.

The solution for the credit good in (2.9) can also be used to solve for optimal labor supply, defining an implicit function,

$$H_t = h(g_t, \theta_t, \mu_{t+1}, \tau_t). \quad (2.13)$$

This equation cannot be solved for H_t explicitly, but the implicit function theorem allows for the construction of an implicit function which defines the explicit function and defined derivatives can be obtained as long as an implicit function is known to exist under the implicit function theorem. Since an implicit function for labor supply can be constructed, the optimal allocation of consumption and labor decisions by households along with real wages are functions of contemporaneous fiscal and monetary policy and the exogenous shocks to government spending and technology.

However, the same is not true for the remaining price system. The equilibrium price level is also dependent on the level of money balances chosen during the previous period which is a result of the cash-in-advance specification. Consequently, the choice of money growth in period t by the government affects the price level in period t and in period $t + 1$. The real and nominal interest rates in period t are also functions of expectations over future government policy, exogenous shocks, and labor supply in period $t + 1$ since the interest rate applied to the stock of real and nominal debt chosen by the household in period t will not be available for use again until period $t + 1$.

The labor supply equation in (2.13) formalizes the assumption of a loss function over distortionary taxes and inflation as discussed in Barro (1979), Barro and Gordon (1981), Bohn (1988), and Schmitt-Grohé and Uribe (2004). These authors use a quadratic loss function to capture the excess burden of taxes and allocative distortions of inflation. The stochastic monetary economy presented here incorporates a loss function in the nonlinearity of the labor supply equation since the contemporaneous tax on labor income and money growth result in direct changes to household labor supply and additional indirect effects through endogenous changes in the marginal product of labor. Taken together, the direct and indirect effects jointly determine optimal household labor supply. While debt is not explicitly present in the labor supply function, it still plays a role since the choices of taxes and money determine the level of debt as a residual in the government budget constraint. Variations in government policy directly affect labor supply, output, remaining household allocations, and the equilibrium price system while feeding back into the government budget constraint through tax revenue. In addition, the shocks to technology and government spending induce responses by both households and the government and will determine the overall volatility of the model economy. Equilibrium decisions by households, firms, and the government are then transmitted across time through the price level and interest rates. Thus, while optimal labor supply is only based on contemporaneous variables, the price system embeds expectations over the future path of policy and the possible realizations of government spending and technology shocks. The degree to which changes in government policy or exogenous shocks impose distortionary effects on equilibrium allocations depends on the amount of nonlinearity present within the system, and within the labor supply function in particular.

3. The Ramsey problem

Real government consumption is assumed to follow an exogenous stochastic process. Households do not receive utility from government spending, but the government must use resources from the production process to finance government consumption. Government policy includes sequences of labor taxes and supplies of money, nominal bonds, and indexed bonds which must satisfy the following budget constraint,

$$\frac{M_t}{P_t} + \frac{B_t^N}{P_t} R_{t-1}^N + B_t^L R_{t-1}^L = \alpha \tau_t (Y_t - X) - G_t + \frac{B_{t+1}^N}{P_t} + B_{t+1}^L + \frac{M_{t+1}}{P_t}, \quad (3.1)$$

where the initial stocks of money, M_0 , nominal bonds, B_0^N , and indexed bonds, B_0^L , are given. The money supply and government spending in period t are assumed to grow at the rate $\exp(\mu_{t+1}) - 1$ and $\exp(g_t) - 1$, respectively. Thus, the level of government spending and money stock are defined as

$$G_t = \exp(g_t) G_{t-1}, \quad (3.2)$$

$$M_{t+1} = \exp(\mu_{t+1}) M_t. \quad (3.3)$$

The random variable g_t is assumed to evolve according to the following autoregressive process,

$$g_t = \rho_g g_{t-1} + \epsilon_{g,t}, \quad 0 < \rho_g < 1, \quad (3.4)$$

where $\epsilon_{g,t}$ is normally distributed with mean zero and standard deviation $\sigma_{g,t}$.^{8,9}

The goal of the government is to maximize the welfare of the household subject to raising revenues through distortionary means. The problem of the government can be modeled as a dynamic programming problem with state variables that include bonds and money balances issued the previous period. After the shocks to the system are revealed, the government selects a policy profile and households respond with a set of allocations. The resulting equilibrium determines the state variables for the next period. Therefore, when choosing an optimal policy mix of taxes, money supply, and debt, the government must take into account the equilibrium reactions by households and firms to the chosen policy mix. The government is also constrained in its policy choices since it must choose a policy mix to maximize household utility while satisfying the government budget constraint. Since the government needs to predict how household allocations and prices will respond to its policy mix, household allocations and prices are defined by rules that associate government policies with allocations. The following definitions describe the Ramsey equilibrium:

Definition 1. A *feasible allocation* is a sequence of $\{C_{1t}\}_{t=1}^{\infty}$, $\{C_{2t}\}_{t=1}^{\infty}$, $\{H_t\}_{t=1}^{\infty}$, $\{G_t\}_{t=1}^{\infty}$ that satisfy the resource constraint in (2.7). A *price system* is a set of nonnegative bounded sequences $\{P_t\}_{t=1}^{\infty}$, $\{w_t\}_{t=1}^{\infty}$, $\{R_t^N\}_{t=1}^{\infty}$, $\{R_t^L\}_{t=1}^{\infty}$. A *government policy* is a set of sequences $\{\tau_t\}_{t=1}^{\infty}$, $\{M_{t+1}\}_{t=1}^{\infty}$, $\{B_{t+1}^N\}_{t=1}^{\infty}$, $\{B_{t+1}^L\}_{t=1}^{\infty}$.

Definition 2. Given the exogenous sequences $\{g_t\}_{t=1}^{\infty}$ and $\{\theta_t\}_{t=1}^{\infty}$; initial stocks of money, nominal bonds, and indexed bonds; and $M_0 = M_0^d$; a *competitive equilibrium* is a feasible allocation, a price system, and a government policy such that (a) given the price system and government policy, the allocation solves both the firm's problem and the household's problem; and (b) given the allocation and price system, the government policy satisfies the sequence of government budget constraints.

Definition 3. The *Ramsey problem* is to choose a competitive equilibrium that maximizes household utility in (2.4). The competitive equilibrium that solves the Ramsey problem is called the *Ramsey plan* or *Ramsey equilibrium*.

The Ramsey problem in the general equilibrium dynamic programming setting incorporates many of the reputational mechanisms for credible government policies as discussed in

⁸For simplicity the model includes the assumption that the correlation between the shock to government spending and the shock to technology are uncorrelated and therefore the income elasticity of demand for government consumption is zero.

⁹The description of government spending in 3.2 and 3.4 is a simplification relative to the more complex fiscal policy appropriations process in place in most countries. The government spending process as modeled here does not include a mechanism by which policymakers and households could smooth policy and allocation decisions based on an expectation of impending changes in government spending. The optimization problem for households and government policymakers described here is therefore an approximation of the true optimization problems which include choosing policy and allocations subject to expectations over building pressures in the appropriations process.

Ljungqvist and Sargent (2000). In general, the government would find it optimal to deviate from its original set of policies if allowed and some mechanism, reputational or otherwise, is needed to ensure credibility of government policy. Under the assumption that an institution or commitment technology exists through which the government can bind itself to a particular sequence of policies, the government attempts to maximize (2.4) subject to (3.1) while taking into account the equilibrium specification for the price system and the optimal response by households.¹⁰ In order to allow for comparison across Ramsey equilibriums with different compositions of nominal and indexed debt, the mix of debt is specified exogenously, leaving the government to choose labor taxes and money growth with the level of debt as the residual.

The government seeks to maximize,

$$V(s_t) = \underset{\Delta_t}{Max} \left\{ \lambda_{gt} \left(\begin{array}{l} a \ln C_{1t} + (1-a) \ln C_{2t} - \gamma H_t + \\ \alpha \tau_t (Y_t - X) - G_t + \frac{B_{t+1}^N}{P_t} + B_{t+1}^L + \\ (\exp(\mu_{t+1}) - 1) \frac{M_t}{P_t} - \frac{B_t^N}{P_t} R_{t-1}^N - B_t^L R_{t-1}^L \end{array} \right) + \beta E_t V(s_{t+1}) \right\},$$

where $s_t = \left(\frac{B_t^N}{P_t}, B_t^L, \frac{M_t^d}{P_t}, \theta_{t-1}, g_{t-1}, \tau_{t-1}, R_{t-1}^N, R_{t-1}^L \right)$ represents the set of state variables, $\Delta_t = (\tau_t, \mu_{t+1}, B_{t+1}^N, B_{t+1}^L)$ is the vector of choice variables, and λ_{gt} is the Lagrange multiplier on the government budget constraint. The first-order conditions for the Ramsey problem are¹¹

$$\tau_t : \left\{ \begin{array}{l} \frac{a}{C_{1t}} \frac{\partial C_{1t}}{\partial \tau_t} + \frac{1-a}{C_{2t}} \frac{\partial C_{2t}}{\partial \tau_t} - \gamma \frac{\partial H_t}{\partial \tau_t} + \\ \lambda_{gt} \left[\alpha \tau_t \frac{\partial Y_t}{\partial \tau_t} + \alpha (Y_t - X) - \frac{B_t^N}{P_t} \frac{\partial R_{t-1}^N}{\partial \tau_t} - B_t^L \frac{\partial R_{t-1}^L}{\partial \tau_t} + \right. \\ \left. \left(\frac{B_t^N}{P_t} R_{t-1}^N - \frac{B_{t+1}^N}{P_t} - (\exp(\mu_{t+1}) - 1) \frac{M_t}{P_t} \right) \frac{1}{P_t} \frac{\partial P_t}{\partial \tau_t} \right] \\ \beta E_t \left\{ \lambda_{gt+1} B_{t+1}^L \frac{\partial R_t^L}{\partial \tau_t} \right\}, \end{array} \right\} = \quad (3.5)$$

$$\mu_{t+1} : \left\{ \begin{array}{l} \frac{a}{C_{1t}} \frac{\partial C_{1t}}{\partial \mu_{t+1}} + \frac{1-a}{C_{2t}} \frac{\partial C_{2t}}{\partial \mu_{t+1}} - \gamma \frac{\partial H_t}{\partial \mu_{t+1}} + \\ \lambda_{gt} \left[\alpha \tau_t \frac{\partial Y_t}{\partial \mu_{t+1}} - \frac{M_{t+1}}{P_t} \exp(\mu_{t+1}) - \frac{B_t^N}{P_t} \frac{\partial R_{t-1}^N}{\partial \mu_{t+1}} - B_t^L \frac{\partial R_{t-1}^L}{\partial \mu_{t+1}} + \right. \\ \left. \left(\frac{B_t^N}{P_t} R_{t-1}^N - \frac{B_{t+1}^N}{P_t} - (\exp(\mu_{t+1}) - 1) \frac{M_t}{P_t} \right) \frac{1}{P_t} \frac{\partial P_t}{\partial \mu_{t+1}} \right] \\ \beta E_t \left\{ \lambda_{gt+1} \left[\left(\frac{B_{t+1}^N}{P_{t+1}} R_t^N - \frac{B_{t+2}^N}{P_{t+1}} \right) \exp(\mu_{t+1}) + B_{t+1}^L \frac{\partial R_t^L}{\partial \mu_{t+1}} \right] \right\}, \end{array} \right\} = \quad (3.6)$$

¹⁰The solution was also solved under discretionary policy. Given that sources of government revenue are distortionary in this economy, optimal policy does not fully inflate away nominal liabilities. This result agrees with Lucas and Stokey (1983) who show that the government has the incentive to inflate away nominal liabilities unless all prices are predetermined or distortionary taxes can be avoided.

¹¹The first order condition for money shown here is actually $\partial/\partial(\exp(-\mu_{t+1}))$. This was done for simplicity of computation. The optimal government policy for money balances can then be found by taking the $-\ln(x)$ of the result.

$$B_{t+1}^N : \frac{\lambda_{gt}}{P_t} = \beta E_t \left\{ \frac{\lambda_{gt+1}}{P_{t+1}} R_t^N \right\}, \quad (3.7)$$

$$B_{t+1}^L : \lambda_{gt} = \beta E_t \left\{ \lambda_{gt+1} R_t^L \right\}, \quad (3.8)$$

where λ_{gt} represents the marginal utility of relaxing the government budget constraint.¹² Investigation of equations (3.5) and (3.6) reveals the importance of maintaining the non-linearity of the labor supply function. The terms representing the impact of labor taxes and money growth include both the direct effects of changes in government policy on labor supply and the indirect effects through changes in the endogenous marginal product of labor. For example, $\frac{\partial C_1}{\partial \tau} = \frac{\partial C_1}{\partial Y} \frac{\partial Y}{\partial H} \frac{\partial H}{\partial \tau}$ and $\frac{\partial C_1}{\partial \mu} = \frac{\partial C_1}{\partial \mu} + \frac{\partial C_1}{\partial Y} \frac{\partial Y}{\partial H} \frac{\partial H}{\partial \mu}$. The direct effect of policy on labor supply is contained in $\partial H / \partial \tau$ and $\partial H / \partial \mu$ and the indirect effect of policy is contained in $\partial Y / \partial H$. The use of linear labor supply and resulting exogenous indirect effects as in Aiyagari et al. (2002), Alvarez et al. (2004), and Schmitt-Grohé and Uribe (2004) eliminates an important channel for optimal household decisionmaking and the evaluation of distortionary government policy. Preserving the endogenous properties of the marginal product of labor is also important in the determination of the variances and covariances of the model economies during simulation.

The Euler condition in (3.5) describes the trade-off between taxation and issuing debt. The first terms on the left-hand side reflect the change in consumption of the cash and credit goods and provision of labor by the household from a change in taxes. A change in the tax rate enters consumption of the cash and credit good indirectly via the equilibrium labor condition. The partial derivative of the equilibrium labor condition with respect to taxes is negative, so that $\partial H_t / \partial \tau_t$, $\partial C_{1t} / \partial \tau_t$, $\partial C_{2t} / \partial \tau_t$ are all negative. Higher labor income taxes discourage labor supply, reduce output, and decrease consumption. The net impact on household utility is negative. The bracketed term in (3.5) describes the change in the government budget constraint from a change in taxes scaled by the multiplier. The first terms inside the bracket represents the direct change in tax revenue from a change in tax policy, the sign of which depends on the nonlinear response of labor supply to a change in taxes. The remaining terms result from the commitment technology and detail the change in the interest rates on nominal and indexed debt during the previous period, respectively, from a change in the one-period ahead tax rate. The next terms inside the bracket describe the price effect on nominal balances and returns on nominal debt. In particular, an increase in taxes increases the price level today since consumption of the cash good falls, reducing the real value of money balances and nominal debt chosen during the previous period. These combined effects in the left-hand side of (3.5) must be equal to the alternative policy of issuing additional debt carried into the next period.

The trade-off between issuing money and debt is more complicated since money enters (3.6) directly through the money growth term and indirectly through the equilibrium labor condition. The first terms on the left-hand side detail the effects of money growth on consumption and labor supply. Increases in money growth decrease equilibrium labor so

¹² Additional terms in (3.5) and (3.6) appear because of the commitment technology in place in relation to the solution under discretionary policy. The household knows the entire sequence while optimizing since the government is committing to a current and future mix of policy across time. Therefore, the additional terms appear since the price system in period t is a function of expected future policy which is known under time commitment.

that $\partial H_t / \partial \mu_{t+1}$ is negative. Combined with the direct effects of money on consumption mean $\partial C_{1t} / \partial \mu_{t+1}$ and $\partial C_{2t} / \partial \mu_{t+1}$ are also negative. The bracketed term, as in the tax derivative, details the impact of changes in money on the government budget constraint scaled by the multiplier. The first term describes the change in labor tax revenue based on the change in equilibrium labor from changes in money growth. Increases in money growth that decrease equilibrium labor result in lower output and reduced labor tax revenue. The second term relates to seigniorage revenues. The next terms arise from the commitment technology and the remaining term describes the price effect on nominal variables. For example, increases in money growth result in a higher price level, reducing the real value of nominal money balances chosen during the previous period. These combined effects on the left-hand side must be equal to the alternative policy of issuing debt which matures during the next period.

3.1. Model calibration and solution procedure

The system of equations that characterize the optimal policies in the Ramsey equilibrium theoretically is nonlinear. Therefore, the system is characterized quantitatively by assigning values to the parameters of technology, spending, preferences, and policy variables. Following the process in Cooley and Prescott (1995), Cooley and Hansen (1991, 1995), Hansen and Wright (1992), Christiano and Eichenbaum (1992), Chari et al. (1991, 1994), and Hansen (1985), the model is calibrated to match the general features of the post-Korean War U.S. economy as reported in the U.S. National Income and Product Accounts (NIPA), modified to account for distortionary taxation and debt composition. The data is used to derive parameter values for the share of income attributable to capital and labor, the capital-output ratio, the fraction of time households spend working in the market, technology and spending shocks, and the ratio of government spending to output. The parameter values are summarized in Table 1.

A gross capital concept is assumed so that investment includes government investment. Hours worked, H , reflects microeconomic studies such as Juster and Stafford (1991) that suggest households spend about a third of their time in work-related activities. Government spending is defined as net real government spending on goods and services, or real total government spending less the sum of real defense investment, real non-defense investment, and real state and local investment. This amount is then taken as a ratio of real net national product. Using quarterly data from 1990:1-2002:4 results in a ratio of 18 percent (the ratio would be several percentage points lower if based on real GDP instead of real net national product). The parameter values for government spending are taken from Hansen and Wright (1992) and Christiano and Eichenbaum (1992), which closely match those reported in Stock and Watson (1999, 31, Table 2). The values for the persistence and standard deviation of the process governing technology are taken from Cooley and Prescott (1995) and Hansen and Wright (1992).

In the non-stochastic steady-state the equilibrium condition for indexed bonds in the household problem equates the real interest rate and the inverse of time preference, β . The quarterly value for the discount factor is set equal to 0.991 which equates to an annual real interest rate of 3.7 percent. This yield closely corresponds to the resulting yield from the initial auctions conducted by the U.S. Treasury for its Treasury Inflation-Indexed Securities

beginning in 1997.¹³ The relative importance of the cash good with respect to the credit good in the utility function is calibrated using first-order conditions from the household problem. In the household problem, first-order conditions can be re-arranged to yield,

$$\frac{C_{1t}}{C_{2t}} = \frac{a}{1-a} [R_{t-1}^N]^{-1}. \quad (3.9)$$

With a binding cash-in-advance constraint, real money balances are used to define C_1 and aggregate consumption minus real money balances is used for C_2 . The nominal interest rate, R^N , is defined by the quarterly return on 3-month Treasury Bills. Taking logs of the above and using ordinary least squares on quarterly data from 1959:1-2002:4 yields an estimate for $a = 0.44$.¹⁴

The initial debt-to-income ratio was calibrated using data on federal government debt held by the public.¹⁵ Since this analysis is interested in debt composition and its associated impacts on policy smoothing, only marketable debt outside the government balance sheet should be considered. Using quarterly data from 1990:1-2002:4 the ratio of federal government debt held by the public to nominal net national product yields a debt-to-income ratio of 0.49. This ratio is used to simulate the U.S. debt-to-income economies or “low” debt economies and then doubled in order to simulate economies with “high” debt. The ratio of inflation-indexed debt to total government debt was about 5 percent, leaving the remaining 95 percent as nominal debt. This analysis however is not solely confined to U.S. characteristics in this regard. For each initial debt stock, the composition is varied from all nominal debt to all indexed debt in order to examine how debt composition influences the Ramsey equilibrium. The remaining variables, γ and δ , are derived from first order conditions and the non-stochastic steady-state government budget constraint.¹⁶

The computational solution procedure is based on the projection approach as described by Judd (1992, 1998) and applied to Ramsey problems in Cosimano and Gapen (2004). The set of Euler conditions from the Ramsey problem, the labor equation from the household’s problem, and the government budget constraint can be generalized to a set of four operator equations $N(f)$ that define equilibrium. Since the set of operator equations is nonlinear, the projection approach begins by defining the policy functions in terms of polynomials. In this case Chebyshev polynomials are used.¹⁷ The solution procedure solves for the optimal

¹³Real yields on U.S. Inflation-Indexed Notes and Bonds have since declined, but current levels may not be indicative of long-term equilibrium real rates of interest on such bonds due to economic and financial market conditions. As such, the rate of time preference was chosen to match earlier data.

¹⁴Given that the results of this regression are sensitive to the time period chosen and accounting for different model specification, the value from this regression is similar to that in Chari et al. (1991).

¹⁵A more accurate measure of debt would include combined federal, state, and local debt in the hands of the public. However, many states have balanced budget amendments which limit the ability of states to issue debt for policy smoothing purposes.

¹⁶The non-stochastic steady-state values for taxes and depreciation used to calibrate the disutility of labor are based on historical U.S. data, including the debt-to-income ratio and composition between nominal and indexed debt. Re-calibration of the model under the various debt and no debt economies simulated in this paper would result in different non-stochastic steady-state values for labor taxes and, in turn, the rate of depreciation and disutility of labor. In order to simulate each economy using constant household preferences, and therefore a constant baseline of preference parameters, the calibrated levels of γ and δ are held constant at their U.S.-based levels across all model economies in this analysis.

¹⁷The boundaries of the space defining the exogenous technology and government spending shocks are

set of policies $(H_t, \mu_{t+1}, \tau_t, \lambda_{gt})$ as functions of the exogenous shocks and state variables that set $N(f) = 0$ simultaneously and satisfy the Ramsey equilibrium. Since the state vector is comprised of information known to all parties at the beginning of the period, the procedure can be viewed as choosing policy functions based on newly revealed information, namely the exogenous shocks to technology and government spending, such that Euler and transversality conditions are satisfied.

The advantage of this approach is that the multiplier from the Ramsey problem, λ_g , is optimally solved for as an endogenous policy variable. The multiplier is the marginal utility of relaxing the government budget constraint by one unit or the value that households place on the ability of the government to raise revenue from a source “outside” the economy - equivalent to collecting a lump-sum tax. Since the projection method is designed to capture higher moments, this process will more accurately illustrate the properties of the multiplier and the cost of distortionary government policy. Consequently, the solution method applied in this paper differs from other recent studies that use a simplified production function (Aiyagari et al. 2002, Alvarez et al. 2004, and Schmitt-Grohé and Uribe 2004) and/or employ the more traditional primal approach (Chari et al. 1994, and Chari and Kehoe 1999).¹⁸ Once properly specified, the system is solved using a nonlinear equation optimizer in Matlab. The results of this procedure are discussed in the next section.

4. The Ramsey equilibrium

The Ramsey problem was solved under various combinations of nominal and indexed debt. For both the low and high-debt economies, the solution was derived under five different combinations of nominal and indexed debt ranging from a policy of indexed debt only (100 percent indexed debt) to a policy of only nominal debt (100 percent nominal debt). Then using the optimal coefficients of the polynomial approximations that describe the Ramsey plan (policy choices of the government, allocations by the household, and the resulting price system), each economy was simulated under the effects of technology and government spending shocks. Statistics were computed by running multiple simulations of 5000 periods in length, taking logarithms, and filtering each simulated time series using the H-P filter (Hodrick and Prescott 1997).

calibrated from actual U.S. data. The interval for each is taken as a multiple of the standard deviation of the error process. Chebyshev collocation methods divide the state space over θ and g into discrete grid points, where higher numbers of points produces a more defined grid space for which the system is solved over. The set of residual functions also contain conditional expectations which must be evaluated. Since the processes that govern the shocks to technology and government spending are assumed to be distributed $N(0, \sigma_{\theta, g}^2)$, expectations can be evaluated using Gauss-Hermite Quadrature. In this procedure, the form of the policy function is assumed to be independent of the realization of the shocks and expectations are found by integrating over the possible realizations of θ and g while treating the policy function as a constant.

¹⁸The primal approach recasts the problem of choosing optimal policy as a problem of choosing allocations subject to constraints which capture restrictions on those allocations. In practice this means using an infinite horizon budget constraint with prices and policy substituted out using first-order conditions, commonly referred to as the implementability constraint. The use of the implementability constraint often requires a search procedure that iterates across candidate solutions for the multiplier (Chari et al. 1994) as opposed to endogenously solving for the multiplier as is done in this paper. Furthermore, the multiplier on the implementability constraint has a different interpretation than the multiplier in this paper.

4.1. Steady-state decision rules

Table 2 presents the steady-state Ramsey equilibrium in levels or growth rates for each model economy. Optimal household allocations smooth consumption and labor supply with the constant a , the relative importance of the cash good to the credit good in the utility function, determining the split between the two consumption goods. In each model economy optimal monetary policy sets money growth equal to the rate of time preference as described in Friedman (1969). In enacting this monetary policy rule the government equates the real gross rate of return across the three assets (money balances, nominal debt, and indexed debt) in expectation, satisfying Euler conditions. As discussed in Alvarez et al. (2004) and Chari et al. (1991, 1996), the so-called Friedman rule turns out to be optimal in a variety of monetary economies with distorting taxes and this paper extends this result to models with different debt composition.

Monetary policy that follows the Friedman rule requires the government to run a gross-of-interest surplus by setting labor income taxes high enough to cover government spending, interest on the debt, and the withdrawal of money balances from the economy. As the debt-to-income ratio rises, the equilibrium tax rate increases to produce a gross-of-interest surplus necessary to cover the associated higher interest costs with the higher stock of government debt. Thus, as steady-state debt levels increase, the distortionary effects of taxation on utility increase and causes lower steady-state labor supply, output, and consumption of both the cash and credit good. The higher welfare costs are illustrated through the value of the multiplier on the government budget constraint which increases when moving from the low-debt to high-debt economy. As debt is introduced for a maintained level of government spending, the shadow value of reducing debt is higher as debt loads increase since distortionary revenue policy imposes additional welfare costs.

4.2. Business cycle moments

Table 3 reports summary statistics on the moments of the business cycle for each model economy. The models only generate about half of the standard deviation of output as found in the U.S. economy, a common shortcoming of most real business cycle models, but generate volatility of consumption, prices, and inflation that more closely match features of U.S. data.^{19,20} Since the price level each period and associated rate of inflation are determined by the cash-in-advance constraint in equilibrium, volatility of the cash good

¹⁹Using historical NIPA data Stock and Watson (1999, 30) report standard deviation of real GDP of 1.66 percent using quarterly filtered data from 1953-1996. Cooley and Prescott (1995, 30) report standard deviation of real GNP of 1.72 percent using quarterly filtered data from 1954:1-1991:2. Some of the reduced model volatility relative to actual U.S. economy characteristics is due to the assumption of a fixed capital stock since standard deviation of investment is much higher than output and consumption. Stock and Watson (1999) report standard deviation of total fixed investment of 4.97 percent, or three times volatility of reported standard deviation of real GDP. Cooley and Prescott (1995) report standard deviation of 8.24 percent for gross private investment and 5.34 percent for total fixed investment. These figures represent five- and three-fold increases over reported standard deviation of GNP.

²⁰Cooley and Prescott (1995) report standard deviation of 1.27 percent and 0.86 percent for total consumption and consumption of non-durables, respectively. Stock and Watson (1999) report standard deviation of consumption (non-durables), the price level, and inflation equal to 1.11 percent, 1.35 percent, and 1.44 percent respectively.

imparts volatility into prices. The volatility of the remaining price system of nominal and real interest rates is based on the filtered value of the gross interest rate series as opposed to a series of net interest rates.²¹ Reported standard deviation of these variables in Table 3 are therefore lower than found in U.S. data on net interest rates.

In addition to the optimality of the Friedman rule, a similar feature of each model is the very low volatility of money growth. Historical standard deviation of the nominal U.S. monetary base is 1.48 (Stock and Watson 1999), yet the models in this paper produce virtually no volatility of money supply. Almost all of the volatility in distortionary revenue generating policy is accounted for through the volatility of labor taxes, which range in standard deviation from 1.65 to 2.85 percent. The volatility of labor taxes in the model however does not appear excessive since it matches closely the estimated standard deviation of the average marginal labor tax rate in the U.S. economy of 2.39 percent as reported by Barro and Sahasakul (1993). The result that labor taxes should fluctuate to preserve stable money growth rates is opposite of Schmitt-Grohé and Uribe (2004) and Chari et al. (1991), who find that money growth should be more variable to preserve smooth taxes on labor income.²²

The properties of each variable under several of the model simulations are displayed in Tables 4 and 5. The cross-correlation of each variable with output, government policy, and exogenous shocks are displayed for both the low and high-debt economies. One major difference between the model economies in this paper and actual U.S. data is the negative correlation between labor and output, which is due to the assumption of a fixed capital stock, eliminating the complementary inputs characteristic of the production function.²³ The existence of capital in the production function combined with the fact that technology augments both capital and labor allows households to balance consumption across both leisure and the two consumption goods, leading to increases in leisure in states of nature that also lead to higher consumption of the cash and credit good. Therefore, positive technology shocks that increase output also increase leisure, and vice versa.

The model economies with all indexed debt produce negative correlations between money growth and output, while the economies with more nominal debt yield a positive correlation. Table 4 reports the cross-correlation between money growth and output under the low debt-to-income ratio and all indexed debt as -0.37 . The correlation changes to 0.38 under a split between nominal and indexed debt and increases to 0.86 under all nominal debt. The economies with higher percentages of nominal debt, therefore, generate a sort of liquidity effect through a negative correlation between money growth and real interest rates.²⁴ In

²¹Gross interest rates are preferable in this case since the steady-state nominal interest rate is equal to 1.00, which makes the net nominal interest rate equal to zero.

²²Schmitt-Grohé and Uribe (2004) study Ramsey policies in a monetary model with sticky prices. When setting parameter values to arrive at a flexible price model and perfect competition, the authors find standard deviation of inflation of 6 percent with standard deviation of taxes of only 0.04 percent. Statistics on money growth are not provided. Chari et al. (1991) examines Ramsey policies in a monetary model without capital similar to the model used in Lucas and Stokey (1983) and find that standard deviation of labor income taxes ranges from 0.06 to 0.11 percent while standard deviation of money growth ranges from 3.74 percent to 54.43 percent, much more volatile than actual data suggests.

²³Cooley and Prescott (1995, 31) report positive correlation between output and total hours of work and average weekly hours of work from using data from both the household survey and establishment survey.

²⁴For examples of other studies on the liquidity effect, see Pagan and Robertson (1995) and Christiano

moving from all indexed debt to the split debt policy, the correlation between money growth and real interest rates declines from 0.68 to 0.11. The correlation turns negative under all nominal debt at -0.42 , indicating the presence of liquidity effects.

The response of government policy, household allocations, and price system to a positive one-period shock to technology and government spending in the low-debt economy with all indexed debt is displayed in Figure 1.²⁵ The behavior of optimal labor supply is complex since household labor supply is a function of the marginal utility of consumption of the credit good, exogenous shocks, government policy, and the marginal product of labor. Changes in government policy and exogenous shocks produce direct effects on labor supply from changes in the after-tax real wage and indirect effects on labor supply from changes in marginal utility of consumption of the credit good. Changes in the real wage cause movement along the upward sloping labor supply schedule while changes in government policy and exogenous shocks that change the marginal utility of the credit good result in shifts of the labor supply schedule.²⁶

A positive shock to technology causes labor supply to increase through the direct effect higher technology has on labor supply through a higher real wage. The same increase in technology, however, also increases overall output. Since additional economy-wide resources are now available, government policymakers can reduce distortionary labor taxes and money growth and still finance the same level of government spending. The reduction in the labor tax rate and money supply have a positive correlation with labor supply that reinforce the direct effect from a higher after-tax real wage since decreases in taxes and money growth increase labor supply.²⁷ However, the increase in technology also decreases the marginal utility of consumption of the credit good, which otherwise causes a decrease in labor supply. Overall, the equilibrium response of household labor supply to a productivity shock depends on the combined directions and magnitudes. In the low-debt economy with all real debt, a positive technology shock causes a slight decline in labor supply in the first period from its steady-state value, resulting in the positive correlation between labor supply and government policy and the negative correlation between labor supply and technology shocks reported in Table 4. The household is able to spread the additional economy-wide resources across both consumption goods and increased leisure since output rises even though labor supply falls. The government is also able to use the additional resources to pay down debt although debt levels are not significantly reduced from their steady-state

(1995).

²⁵Each set of vertical panels in the figure reports the percentage deviation from steady-state values for the relevant variables under a positive one-standard deviation shock to technology (left vertical panels) and government spending (right vertical panels). As in Table 2, percentage deviation of real and nominal interest rates are based on gross rates. Deviation of money growth is based on the net money growth rate. The cross-correlations from the simulations in Tables 4 and 5 are based on filtered data as opposed to the impulse response functions displayed in Figure 1 which are based on raw data. The use of the H-P filter generally reduces the persistence of the various series (i.e. reduces the tendency for the variables to remain away from their steady-state values) and occasionally changes the sign of the initial response if the percentage deviation under raw data is very low. Nevertheless, this section proceeds with the standard use of raw data since the exercise remains illustrative of model relationships.

²⁶See appendix for properties of labor supply and labor demand.

²⁷With respect to government policy, the implicit function labor supply equation indicates household labor supply is negatively related to both labor taxes and money supply so that $\partial H_t / \partial \tau_t < 0$, $\partial H_t / \partial \mu_{t+1} < 0$. See Appendix for defined derivatives under the implicit function theorem.

levels. The reduction in distortionary labor taxes and monetary policy, along with slight declines in outstanding debt, result in a lower value for the multiplier on the government budget constraint. In a situation where additional economy-wide resources are available, the shadow value of reducing debt and the marginal cost of financing government spending have been reduced.

The effect of the positive shock to technology on prices is dependent on the change in the level of consumption of the cash good since the price level is determined through the cash-in-advance constraint which holds with equality in equilibrium. In this case a higher level of consumption of the cash good lowers the period t price level relative to its steady-state value since nominal money balances were chosen during period $t - 1$ for use in period t . However, in period $t + 1$ onward the positive technology shock results in higher inflation relative to steady-state values since consumption of the cash good begins to return to its steady-state level, or $C_{1t+i+1} < C_{1t+i}$, and offsets the lower money growth rate. Consequently, the inflation dynamics in response to a positive technology shock first results in lower inflation in the initial period of the shock and then slightly higher inflation relative to steady-state inflation as the shock begins to expire.

With respect to the remaining price system, both nominal and real interest rates decline from the positive technology shock. The nominal interest rate declines in the first period since the positive technology shock allows for a reduction in money growth. Since a reduction in money growth implies a higher rate of withdrawal of nominal money balances, the one-period nominal interest rate that prevails in period t falls below unity since expectations of money growth in period $t + 1$ remain below the steady-state level of money growth. The one period real interest rate falls in period t since the expected marginal value of consumption of the credit good in period $t + 1$ is less than the level that prevails in period t as a result of the technology shock. The path that consumption of the credit good takes in return to the steady-state combined with Jensen's inequality effects results in a decline in real interest rates.

A positive shock to government spending is displayed in the right column of Figure 1. A positive shock to government spending causes labor supply to decrease through the direct effect higher taxes has on labor supply through a lower after-tax real wage. The increase in labor taxes coincides with increases in money growth and slight increases in debt since policymakers need to finance the additional government spending. The increase in the labor tax rate and money supply have a negative effect on labor supply that reinforce the direct effect from a lower after-tax real wage since increases in taxes and money growth decrease labor supply through the implicit function governing labor supply. However, the increase in government spending also increases the marginal utility of consumption of the credit good, which otherwise induces an increase in labor supply. In the low-debt economy with all real debt, these effects are largely offsetting, causing negligible declines in labor supply and output.²⁸ The resulting lack of correlation between shocks to government spending and both labor supply and output in the low-debt economy with all indexed debt are reflected in Table 4.

Since output remains essentially flat, the increased government spending pulls economy-wide resources away from the household, resulting in reduced consumption of both cash

²⁸The decline in labor supply is -0.002 percent while the decline in output is -0.001 percent.

and credit goods while leisure remains relatively unchanged. The increase in distortionary labor taxes and money growth, along with slight increases in outstanding debt, result in a higher value for the multiplier on the government budget constraint. In a situation where additional government spending makes claims on an unchanged amount of economy-wide resources, the shadow value of reducing debt and the marginal cost of financing government spending have been increased. In these states, the household would value the ability of the government to collect a tax from “outside” the economy (i.e. a lump-sum tax), thereby reducing the distortionary impacts of revenue generating policy. This is reflected in a higher value of the multiplier on the government budget constraint which increases 3 percent from its steady-state level in the same period as the positive shock to government spending is revealed.

The positive shock to government spending displays the expected positive relationship on prices. A lower level of consumption of the cash good increases the period t price level since nominal money balances have already been chosen during the previous period. In contrast to the positive technology shock, inflation remains above its steady-state level while the government spending shock persists. From period $t + 1$ onward, $C_{1t+i+1} > C_{1t+i}$ which otherwise reduces inflation, but this effect is offset by higher money growth leaving inflation slightly above steady-state inflation for the duration of the government spending shock. With respect to the remaining price system, both nominal and real interest rates increase as a result of the increase in government spending. The nominal interest rate increases in the first period since the government spending shock results in an increase in money growth. Expectations of money growth in period $t + 1$ remain above the steady-state rate of money growth. The one period real interest rate rises in period t since the expected value of consumption of the credit good in period $t + 1$ is more than the level that prevails in period t as consumption begins to return to steady-state levels.

The behavior of optimal taxes and debt in the model economies simulated in this paper resembles that in Lucas and Stokey (1983) and Chari et al. (1991) since neither taxes or debt exhibit random walk behavior. The Ramsey problem formulated by Lucas and Stokey (1983) has complete markets, flexible prices, no capital, exogenous Markov government expenditures, state-contingent taxes, and government debt. They found that tax smoothing in the context of lower variance of taxes relative to a strict balanced budget rule, and that taxes should generally inherit the serial correlation of government spending. Chari et al. (1991) use a similar model to show that the Ramsey allocation in a flexible price economy with nominal debt that is state-contingent in real terms behaves in a similar fashion. They find that autocorrelations of the inflation rate are small or negative in the economies with money and therefore inflation and money growth are not random walks.²⁹

The non-random walk behavior of policy stands in contrast to Barro (1979), Aiyagari et al. (2002), and Schmitt-Grohé and Uribe (2004). Both Barro (1979) and Aiyagari et al. (2002) find that debt and taxes should follow random walks under incomplete markets and only risk-free government debt. Aiyagari et al. (2002) retains the general features of Lucas and Stokey (1983) - no capital, linear labor supply, exogenous government spending - but modifies the bond structure to only include real one-period government debt, similar to the

²⁹See Mankiw (1987) for a partial equilibrium formulation of why inflation should follow a random walk process.

indexed debt used in this paper. They also restrict utility to be linear in consumption and concave in leisure to more closely match the intuition in Barro (1979).³⁰ Schmitt-Grohé and Uribe (2004) relax the assumption of flexible prices and find that debt and taxes display random walk properties under a limited degree of price stickiness.

The model economies in this paper differ from the Lucas and Stokey (1983) framework and alternative specifications in Aiyagari et al. (2002) and Schmitt-Grhoé and Uribe (2004) in that they contain a fixed capital stock and nontrivial nonlinear labor supply. However, even with these differences, the models simulated in this paper produce non-random walk behavior of taxes, debt, and inflation. This is true even in the model economies with all nominal debt that is state-contingent in real terms. While Ramsey allocations retain non-random walk properties regardless of debt composition, some of the correlations between variables and impulse response functions detailed above under all indexed debt change as debt composition is varied. For example, the directional response in the first and second period for household allocations, government policy, and the price system to uncertainty using the unfiltered data is displayed in Table 6. Changes in the directional response of the multiplier, money growth, labor taxes, labor supply, and inflation are all visible from the table. Furthermore, even though the directional response by some of the variables to uncertainty do not change when moving from indexed to nominal debt, the magnitude of the correlation may change. The only exogenously calibrated difference between the two model economies in the table is that one contains all indexed debt and the other contains all nominal debt. All other variables remain calibrated as described in the model setup and solution procedure. The next section discusses both the impact of debt and the value of nominal debt in the Ramsey problem, including reasons for the changing directional responses in Table 6.

4.3. The hedging role of nominal debt

As indicated in Table 2 and Figure 2, business cycle volatility is reduced in economies with higher percentages of nominal debt. Economies with higher ratios of nominal debt are less volatile than economies with higher ratios of indexed debt since nominal debt acts as a hedge against shocks to the government budget. Unexpected shocks to the economy that call for an increase in government revenue (i.e. higher labor taxes or money growth) also correspond to states with higher inflation. This higher-than-expected inflation reduces the value of pre-existing nominal debt and balances the need to increase government revenue. Nominal debt therefore acts as a hedge against shocks to the government budget by providing a non-distortionary source of revenue, allowing for a smoother path for distorting revenue generating policies and consumption. In contrast, the unexpected component of inflation does not affect the real value of indexed debt which is designed to provide a constant real rate of return over the period.

Nominal debt plays a hedging role when the economy faces a positive shock to government spending. A positive shock to government spending causes optimal government policy to respond with an increase in labor taxes and money growth, as shown by the positive cor-

³⁰Aiyagari et al. (2002) also includes natural limits on debt which are unnecessary for the model in this paper since the utility function and model specification assure that transversality conditions hold.

relation between government spending shocks and the labor tax rate and money growth in each of the economies with debt in Tables 4 and 5. However, government spending shocks are also positively correlated with the price level and inflation. As the government consumes more output, households generally respond by increasing labor supply, but the increase in output is not enough to offset the total increase in government spending and consumption of the cash good falls.³¹ Since they reduce consumption of the cash good relative to previously chosen money balances, positive shocks to government spending result in a higher price level and higher-than-expected inflation which reduces the real value of nominal debt issued during the previous period. The reduction in the real value of nominal debt counterbalances the need for an increase in labor taxes and money growth. This effect is clearly illustrated through the decreased correlation between taxes and money growth relative to shocks to government spending as the ratio of nominal debt increases. The correlations between government spending shocks and labor taxes and money growth are displayed in the bottom panels of Figure 3 for different combinations of nominal and indexed debt. Under both the low and high debt-to-income ratios the correlation between shocks to government spending and these government policy variables are lowest when the percentage of nominal debt is highest.

A similar hedging role is present with respect to negative technology shocks. Beginning in both economies with all indexed debt, a negative one-period shock to technology causes output to decrease through the production function even though the household responds by increasing labor supply. The decrease in output creates a need for higher labor taxes and money growth to finance the same level of government spending. However, as was the case with positive government spending shocks, the negative technology shock is positively correlated with the price level and inflation. The correlation between the price level and technology shocks is consistently high across each economy with debt in Tables 4 and 5. A negative technology shock results in a decline in consumption of the cash good and this causes an increase in the price level and inflation through the equilibrium cash-in-advance constraint. The higher-than-expected inflation reduces the real value of existing nominal debt and offsets the need to raise the distortionary labor tax rate and money growth.

Nominal debt is a valuable hedge in the two scenarios described above since the unexpected inflation occurs in states of the world that would otherwise call for increases in distortionary revenue policy.³² Since the hedging role of nominal debt occurs when the government is either consuming more of existing economy-wide resources (positive government spending shock) or maintaining existing levels of spending when economy-wide resources are reduced (negative technology shock), the marginal cost of fiscal and monetary policy on household welfare increases. Consequently, the role of nominal debt as a hedge is most clearly illustrated through a positive correlation between inflation and the multiplier on the

³¹In the economies with all indexed debt, the correlations between shocks to government spending and labor supply are essentially zero. The correlation under low debt is 0.01 and under high debt (0.02). As nominal debt is added, the correlation turns consistently positive under both low and high debt. However, the correlation between government spending shocks and consumption of the cash good is negative across all economies, regardless of debt composition. Therefore, shocks to government spending are positively correlated with the price level and inflation.

³²While this result confirms the original conjecture by Bohn (1988), the analysis here is done in a fully-specified general equilibrium setting.

government budget constraint. In these states of the world - positive shocks to government spending and negative shocks to technology - the shadow value of reducing debt is increased, resulting in a positive correlation between inflation and the multiplier. The reduction in the real value of pre-existing nominal debt provides a non-distortionary channel of revenue in the government budget constraint, easing the welfare costs of distortionary policy.

Nominal debt as a shock absorber, however, works in the opposite direction under positive technology shocks and negative government spending shocks. These states of the world produce lower-than-expected inflation which increases the real value of pre-existing nominal debt and otherwise calls for increased labor taxes and money growth to finance the higher debt costs. Yet these increased debt service costs occur when additional resources become available, either through additional output from higher technology or less government consumption. With additional economy-wide resources available to the household under both cases, the household can absorb these additional debt costs more easily since the marginal cost of distortionary government policy has fallen. In the case of a positive technology shock and high levels of nominal debt, government policy calls for higher labor taxes and money growth, but sufficient economy wide resources remain available from higher output to increase consumption and leisure while simultaneously decreasing the stock of debt. Under negative government spending shocks, government policy reduces labor taxes and money growth, allowing for a similar pattern across consumption, leisure, and debt, but in lower magnitudes. Thus, even though the multiplier and inflation are negatively correlated in these circumstances, suggesting that indexed debt would be preferable to nominal debt, sufficient resources are available to more than offset any negative welfare implications.³³ Overall, the value of nominal debt as a hedge occurs when needed most.

Nominal debt will have value as a hedge as long as the correlation between the multiplier on the government budget constraint (i.e. the shadow value of reducing debt) and inflation is positive. The optimal level of nominal debt, as derived from its role as a hedge against shocks to the government budget, is directly dependent on the type and size of the shocks that hit the economy and the amount to which the household values stability of policy. Since the household prefers smooth consumption and leisure, the optimal amount of nominal debt would sufficiently counteract the direct effect of a shock to the government budget and mute any distortionary policy response. As the amount of nominal debt increases the marginal value of the gain in hedge falls since the absolute volatility of the system is reduced. The simulations of the low and high debt economies indicate that the correlation between the multiplier and inflation is highest under the economies with all indexed debt, indicating that existing hedge opportunities are not being filled. The correlation between the multiplier and inflation is plotted in the top panel of Figure 3 under each of the simulated debt characteristics. As the percentage of nominal debt is increased, the correlation begins to decline as the most valuable hedging opportunities are filled. Under the low-debt economy with all nominal debt, the correlation between the multiplier and inflation falls to 0.01, suggesting that hedging opportunities of nominal debt are nearly exhausted. This indicates

³³Shocks to government spending and technology in period t are revealed at the beginning of the period, but after debt levels and composition were chosen during the previous period. Therefore, since debt is chosen before the realization of shocks, nominal debt is strictly preferable to indexed debt until optimal levels of hedging have been achieved. If government policymakers knew the distribution of shocks ahead of time, debt composition could be matched more explicitly with the shocks.

that the current U.S. debt level with 95 percent nominal debt and 5 percent indexed debt is roughly optimal from a hedging perspective. Under the high debt case, the value of nominal debt as a hedge is maximized at around 65 percent nominal debt, 35 percent indexed debt. Thus, while Aiyagari and McGrattan (1998) find that the current level of U.S. debt is optimal, this study indicates that the current composition between nominal and indexed debt is also optimal.³⁴

4.4. Measuring the gains

A utility equivalence framework is used to measure the gain to households from the reduction of policy and economic instability through the hedging role of nominal debt. The utility equivalence is measured as the lump-sum present discounted value of utility that makes the household indifferent between the given debt level with all indexed debt and the selected proportion of nominal debt. Since the household has three variables that enter into the utility function, utility equivalent measures were computed for each variable, thereby highlighting the contribution that each plays in utility gains. Values were taken from an average across a similar set of simulations for each calibrated economy and are displayed in Table 7. As discussed in Kim and Kim (2003), the size of welfare gains in this paper are also dependent on preserving nonlinearities within the model as opposed to alternative methods, such as loglinearization, which either underestimates gains or introduces errors in the estimation procedure.

The table displays the gain in lifetime utility from moving from one debt composition to another under the same debt-to-income ratio (e.g. the value of nominal debt as a hedge). For example, the gain in lifetime utility from moving from all indexed debt to a combination of 25 percent nominal debt and 75 percent indexed debt under the low-debt economy is 0.4 percent. Moving from an indexed debt structure to a fully nominal debt structure in the low-debt economy yields a gain in lifetime utility of 0.7 percent. As indicated above for the high debt specification, gains in lifetime utility from nominal debt as a hedge is maximized at 65 percent nominal debt and 35 percent indexed debt. Gains decline thereafter as higher percentages of nominal debt are used. As shown in the table, the gain in lifetime utility from nominal debt as a hedge is 0.7 percent under a split debt policy (50 percent nominal debt, 50 percent indexed debt) and then declines to 0.4 percent under all nominal debt. Though the full nominal debt structure remains optimal relative to the all indexed debt structure, gains in lifetime utility would be maximized by employing a mixture of more indexed debt and less nominal debt.

The suggested welfare gains from the hedging role of nominal debt are significant in that they are roughly equivalent in magnitude to the gains from reducing business cycle volatility. Lucas (1987, 20-31) estimated the cost of business cycles as the percentage increase in consumption required to leave the consumer indifferent between consumption volatility based on actual data and a perfectly smoothed series of consumption. Using logarithmic preferences and post World War II data series, Lucas reports that completely

³⁴ Aiyagari and McGrattan (1998) find that the optimal level of debt is two-thirds of GDP, which is equal to the average debt/GDP ratio in the post-war U.S. economy. However, their analysis includes both U.S. federal and U.S. state debt. Eliminating U.S. state debt from this total corresponds to the debt to income ratio used to calibrate the U.S. debt ratio in this paper.

removing consumption variability entails a lifetime increase in utility equal to 0.2 percent of consumption. However, Lucas uses a time preference parameter of $\beta = 0.95$ and using a calibrated value equal to $\beta = 0.991$ as in this paper would result in a lifetime increase in utility equal to 0.9 percent. Employing the optimal percentage of nominal debt for hedging purposes in the low and high-debt economies in this paper results in gains in lifetime utility of 0.7 percent, yet consumption and output volatility are only reduced between one-third and one-half their values under all indexed debt relative to the full volatility reduction estimated in Lucas (1987).

The welfare gains from hedging with nominal debt are also similar in size to the gains from eliminating moderate inflation as reported by Cooley and Hansen (1991, 497-498) and Aiyagari et al. (1998, 1294-1299), who estimate the gains from eliminating moderate inflation. Employing a similar stochastic monetary economy as is used in this paper, Cooley and Hansen (1991) report that transitioning from 5 percent and 10 percent inflation to zero inflation results in gains in lifetime utility of 0.4 and 0.6 percent, respectively. Aiyagari et al. (1998) examine the relationship between the size of the banking sector and inflation and estimate that the welfare cost of inflation is 0.5 percent of consumption after accounting for transitional dynamics.

The results in this paper also counter two main arguments in Schmitt-Grohé and Uribe (2004), namely that (i) the government faces a trade-off between price stability and the use of unexpected inflation as a non-distortionary tax on household wealth, and (ii) the welfare gains from unexpected inflation are very small. These authors contrast a flexible price economy where optimal policy yields a volatile inflation process versus an economy with price stickiness where optimal policy minimizes inflation volatility.³⁵ They conclude that a small amount of price rigidity, only around one-tenth the estimated degree of price stickiness in the U.S. economy, is necessary to make policymakers choose price stability over the unexpected inflation tax and consequently the gains from the latter must be small. Aiyagari et al. (2002) make a related claim since they report virtually no difference in welfare between economies with and without state-contingent debt. The results in this paper indicate however that government policymakers face no such tradeoff. Policymakers can target price stability and utilize the hedging role of nominal debt in a meaningful way. As the simulation results indicate, volatility of inflation in each of the model economies remains low while the estimated welfare gains from the hedging properties of nominal debt are significant. The government is able to maintain very low volatility of money growth and therefore preserve the Friedman rule, while fluctuations in the cash good impart most of the volatility into prices through the cash-in-advance constraint. Capturing the higher moments within the model, and within the nonlinear labor supply function in particular, is essential in measuring overall risk and the cost of distortionary policy. This paper therefore lends additional support to Kim and Kim (2003) who show that traditional log-linearization techniques may not produce accurate welfare computations since it neglects higher moments.³⁶

³⁵The authors report standard deviation of inflation of 7 percent per year under flexible prices and perfect competition versus 0.17 percent under the baseline sticky-price economy (Schmitt-Grohé and Uribe 2004, 214).

³⁶The use of linear labor supply by Schmitt-Grohé and Uribe (2004) may also be the reason that the authors report little quantitative difference between their log-linear approximation and log-quadratic approximation.

5. Conclusion

This paper focuses on the importance of debt composition in the setting of optimal fiscal and monetary policy over the business cycle. The ability to issue debt or deepen access, even if indexed, should entail gains through the additional degree of freedom for government policymakers versus little or no ability to issue debt. However, this analysis suggests that a predominantly indexed debt structure is a second-best solution. Fixed-rate nominal debt that acts as state-contingent debt in ex-post real returns can be a significant tool to reduce volatility of distortionary government policy since such debt acts as a hedge against unexpected shocks to the budget. Optimal debt policy should include sufficient amounts of nominal debt to further smooth distortionary government policy and reduce macroeconomic and business cycle volatility.

However nominal debt does not strictly dominate real debt and the results of this paper should not be interpreted as suggesting more nominal debt is always better. This analysis shows there are limits to the hedging properties of nominal debt with the correlation between inflation and the multiplier on the government budget constraint as the critical determinant of optimal debt composition. The ability to solve for the multiplier as an endogenous variable should therefore be viewed as a main advantage of the approach taken in this paper. Policymakers also need to balance any expected gains from the hedging effects of nominal debt with the steady-state effects of debt. A higher tax-transfer burden is imposed by a higher level of government debt in relation to output. Having a lower debt-to-income ratio always allows policymakers as a practical matter to respond to shocks at least as optimally they could under a higher debt-to-income ratio since there is unused borrowing capacity. This paper does not address the issue of actual borrowing capacity and the exogenous levels of debt-to-income were chosen as a reflection of current U.S. policy and to illustrate the role of nominal debt as a hedge.

In a broader context this paper lends additional credence to the argument that economic growth and macroeconomic volatility are negatively related, and that reductions in macroeconomic volatility and minimization of the cost of business cycles can entail increases in overall welfare. While this stochastic model economy examines convexities from distortionary taxation and money growth on labor supply preferences, many other examples of convexities can be found in existing studies. The idea that government policy uncertainty could have negative effects on growth was examined by Aizenman and Marion (1993) who find that the magnitude and persistence of tax policy fluctuations jointly determine the pattern of investment and growth with negative correlation. Analysis by Bernanke (1983) on irreversible investment and by Ramey and Ramey (1991) on rigidities in the production process suggest that increased volatility results in lower investment and, therefore, lower growth. Black (1987) examined whether countries face a choice between a high-growth, high-variance economy and a low-growth, low-variance economy depending on the available technology, suggesting the growth and volatility may be positively correlated. More recently, Ramey and Ramey (1995) examine cross-country data and find that reductions in the volatility of output growth equal to one standard deviation of its cross-country variation equates to an increased growth rate of one-third of one percent in OECD countries, roughly equal to the increases in steady-state output and consumption reported here.

One natural extension of this line of research would be to conduct similar analysis to

include foreign currency denominated debt in an open-economy setting. Foreign currency denominated debt has played a major role in many recent emerging market crises, causing large adjustments in the public sector balance sheet from sudden exchange rate depreciations. Such events have often resulted in pro-cyclical fiscal policy as the public sector is forced to raise revenue during an economic downturn to stay current on its debt obligations. Extrapolating from the current model framework, it is likely that the gains from domestic currency nominal debt as a hedge would be larger under these circumstances than those calculated in this paper. We therefore surmise that the gains reported here offer a lower bound on the gains from optimal debt composition. Moving to an open-economy setting however is not trivial and is left for future research.

6. Appendix: Household optimization problem

This Appendix details the solution to the household optimization problem. The household chooses consumption of the cash and credit goods, the amount of money to be carried into the next period, and stocks of nominal and indexed government debt to maximize (2.4) subject to the budget constraint in (2.5) and the cash-in-advance constraint in (2.6). This can be set up as a dynamic programming problem,

$$V(s_t) = \underset{\Delta_t}{Max} \left\{ \begin{array}{l} a \ln C_{1t} + (1-a) \ln C_{2t} - \gamma H_t + \lambda_{1t} \left(\frac{M_t^d}{P_t} - C_{1t} \right) + \\ \lambda_{2t} \left((1 - \alpha\tau_t)(Y_t - X) + \frac{M_t^d}{P_t} + \frac{B_t^N}{P_t} R_{t-1}^N + B_t^L R_{t-1}^L \right) + \beta E_t V(s_{t+1}) \end{array} \right\},$$

where $s_t = \left(\frac{B_t^N}{P_t}, B_t^L, \frac{M_t^d}{P_t}, \theta_{t-1}, g_{t-1}, \tau_{t-1}, R_{t-1}^N, R_{t-1}^L \right)$ is the set of state variables and $\Delta_t = (C_{1t}, C_{2t}, M_{t+1}^d, B_{t+1}^N, B_{t+1}^L, H_t)$ is the vector of choice variables. Here, λ_{1t} and λ_{2t} are the Lagrange multipliers for the cash-in-advance constraint and household budget constraint, respectively. The resulting first-order conditions are,

$$C_{1t} : \frac{a}{C_{1t}} = \lambda_{1t} + \lambda_{2t}, \quad (6.1)$$

$$C_{2t} : \frac{1-a}{C_{2t}} = \lambda_{2t}, \quad (6.2)$$

$$M_{t+1}^d : \frac{1-a}{C_{2t}} = \beta E_t \left\{ \frac{\lambda_{1t+1}}{P_{t+1}} + \frac{\lambda_{2t+1}}{P_{t+1}} \right\}, \quad (6.3)$$

$$B_{t+1}^N : \frac{\lambda_{2t}}{P_t} = \beta E_t \left\{ \frac{\lambda_{2t+1}}{P_{t+1}} R_{t+1}^N \right\}, \quad (6.4)$$

$$B_{t+1}^L : \lambda_{2t} = \beta E_t \left\{ \lambda_{2t+1} R_{t+1}^L \right\}, \quad (6.5)$$

$$H_t : \gamma = \lambda_{2t} (1 - \alpha\tau_t) \alpha \frac{Y_t}{H_t}. \quad (6.6)$$

Here the Benveniste-Scheinkman condition is used repeatedly. Combining the first-order condition on the cash good in (6.1) with the first-order condition on labor in (6.6) yields

$$\lambda_{1t} = \frac{a}{C_{1t}} - \frac{\gamma}{(1 - \alpha\tau_t)} \frac{H_t}{\alpha Y_t}. \quad (6.7)$$

The multiplier on the cash-in-advance constraint is equal to the marginal utility of consumption of the cash good reduced by the marginal disutility of having to supply additional hours of labor for the equal amount of consumption of the credit good. Combining the first-order condition for money balances in (6.3) and the first-order condition for nominal bonds in (6.4) and defining the gross real return on money balances as the inverse of the gross rate of inflation, or $R_t^M = P_t/P_{t+1}$, yields

$$\lambda_{2t} \left(\frac{R_t^N - 1}{R_t^N} \right) = \beta E_t \left\{ R_t^M \left(\frac{a}{C_{1t+1}} - \frac{\gamma}{(1 - \alpha\tau_{t+1})} \frac{H_{t+1}}{\alpha Y_{t+1}} \right) \right\}. \quad (6.8)$$

The left hand side of the above is the utility cost of holding money balances instead of nominal bonds. $R_t^N - 1$ is the lost nominal interest earnings in period $t + 1$ discounted to time t by the nominal interest rate and expressed in utility through λ_{2t} . This must be equal to the expected value at time t of carrying money balances into period $t + 1$. A similar Euler condition in real terms can be derived using the condition on money balances in (6.3) with the condition on indexed bonds in (6.5). Doing so results in

$$\lambda_{2t} \left(\frac{R_t^L - E_t R_t^M}{R_t^L} \right) = \beta E_t \left\{ R_t^M \left(\frac{a}{C_{1t+1}} - \frac{\gamma}{(1 - \alpha\tau_{t+1})} \frac{H_{t+1}}{\alpha Y_{t+1}} \right) \right\}. \quad (6.9)$$

The left hand side of (6.9) is the utility cost of holding money balances instead of indexed bonds. $R_t^L - E_t R_t^M$ is the lost real interest earnings in in period $t + 1$ discounted to time t by the real interest rate and expressed in utility terms. This quantity must be equal to the expected value of carrying money balances into period $t + 1$.

The first-order conditions in (6.1)-(6.6) can be combined to form the following Euler conditions,

$$M_{t+1}^d : \frac{1 - a}{C_{2t}} = \beta E_t \left\{ \frac{a}{C_{1t+1}} \frac{P_t}{P_{t+1}} \right\}, \quad (6.10)$$

$$B_{t+1}^N : \frac{1 - a}{C_{2t}} = \beta E_t \left\{ \frac{1 - a}{C_{2t+1}} \frac{P_t}{P_{t+1}} R_t^N \right\}, \quad (6.11)$$

$$B_{t+1}^L : \frac{1 - a}{C_{2t}} = \beta E_t \left\{ \frac{1 - a}{C_{2t+1}} R_t^L \right\}, \quad (6.12)$$

$$H_t : \gamma C_{2t} = (1 - a) (1 - \alpha\tau_t) \alpha \frac{Y_t}{H_t}. \quad (6.13)$$

The Euler conditions on nominal and indexed bonds can be used to derive the conditions on the two interest rates as

$$R_t^L = \frac{1}{\beta C_{2t}} \left[\frac{1}{E_t \left[\frac{1}{C_{2t+1}} \right]} \right], \quad (6.14)$$

$$R_t^N = \frac{1}{\beta C_{2t}} \left[\frac{1}{E \left[\frac{1}{C_{2t+1}} \frac{P_t}{P_{t+1}} \right]} \right]. \quad (6.15)$$

Maximization of expression (2.4) is subject to $M^d \geq 0$ for all $t \geq 0$, given the initial stock of money, M_0 . There is no similar restriction on debt since negative stocks of government bonds would indicate household indebtedness to the government, although transversality conditions will prevent debt from growing without bound in either direction. Transversality conditions can be derived by consolidating two consecutive household budget constraints

yielding

$$\begin{aligned}
& C_{1t} + C_{2t} + \frac{1}{R_t^L} (C_{1t+1} + C_{2t+1}) + \tag{6.16} \\
& \frac{M_{t+1}^d}{P_t} \left(1 - \frac{1}{R_t^L} \frac{P_t}{P_{t+1}} \right) + \frac{B_{t+1}^N}{P_t} \left(1 - \frac{R_t^N}{R_t^L} \frac{P_t}{P_{t+1}} \right) \\
\leq & (1 - \alpha\tau_t)(Y_t - X) + \frac{M_t^d}{P_t} + \frac{B_t^N}{P_t} R_t^N + B_t^L R_t^L + \\
& \frac{1}{R_t^L} \left[(1 - \alpha\tau_{t+1})(Y_{t+1} - X) - \frac{M_{t+2}^d}{P_{t+1}} - \frac{B_{t+2}^N}{P_{t+1}} - B_{t+2}^L \right].
\end{aligned}$$

To ensure a bounded budget set, the term multiplying M_{t+1}^d/P_t must be greater than or equal to zero. If this was not the case, households could make infinitely large profits by increasing money balances financed by issuing bonds. Since money balances earn no interest, the gross real return on money from t to $t+1$ is just the inverse of the inflation rate, or $R_t^M = P_t/P_{t+1}$. The result is that real return on money must be less than or equal to the return on bonds,

$$1 - \frac{1}{R_t^L} \frac{P_t}{P_{t+1}} = 1 - \frac{R_t^M}{R_t^L} \geq 0, \tag{6.17}$$

or the net nominal interest rate cannot be negative.

If the process of recursively using successive household budget constraints to eliminate successive indexed bond terms is continued, the present-value budget constraint of the household can be derived as

$$\begin{aligned}
& \sum_{i=0}^{\infty} q_i \left[\begin{aligned} & C_{1t+i} + C_{2t+i} + \frac{M_{t+i+1}^d}{P_{t+i}} \left(1 - \frac{1}{R_{t+i}^L} \frac{P_{t+i}}{P_{t+i+1}} \right) \\ & + \frac{B_{t+i+1}^N}{P_{t+i}} \left(1 - \frac{R_{t+i}^N}{R_{t+i}^L} \frac{P_{t+i}}{P_{t+i+1}} \right) - (1 - \alpha\tau_{t+i})(Y_{t+i} - X) \end{aligned} \right] \tag{6.18} \\
\leq & \frac{M_t^d}{P_t} + \frac{B_t^N}{P_t} R_{t-1}^N + B_t^L R_{t-1}^L,
\end{aligned}$$

where

$$q_0 = 1 \text{ and } q_i = \prod_{n=1}^i \frac{1}{R_{t+n-1}^L}, \tag{6.19}$$

and where the following transversality conditions have been imposed,

$$\lim_{I \rightarrow \infty} (q_I B_{t+I+1}^L) = 0, \tag{6.20}$$

$$\lim_{I \rightarrow \infty} \left(q_I \frac{B_{t+I+1}^N}{P_{t+I}} \right) = 0, \tag{6.21}$$

$$\lim_{I \rightarrow \infty} \left(q_I \frac{M_{t+I+1}^d}{P_{t+I}} \right) = 0. \tag{6.22}$$

Households would not find it optimal to accumulate levels of money balances, indexed bonds, or nominal bonds that violate these conditions because alternative allocations exist that would afford higher levels of consumption and higher lifetime utility.

The specification of log preferences allows for the derivation of closed-form solutions for consumption, prices, and interest rates since the income and substitution effects cancel. First, substitute the cash-in-advance constraint in (2.6) and (3.3) into the Euler condition for money balances in (6.10) to solve for the ratio of consumption of the cash good to consumption of the credit good. Assuming that $M_{t+1} = M_{t+1}^d$ in equilibrium,

$$\frac{C_{1t}}{C_{2t}} = \beta \left(\frac{a}{1-a} \right) \exp(-\mu_{t+1}). \quad (6.23)$$

The resource constraint in (2.7) can then be used with the above to calculate the closed-form solutions for consumption,

$$C_{1t} = \frac{(Y_t - X - G_t) \beta \left(\frac{a}{1-a} \right) \exp(-\mu_{t+1})}{1 + \beta \left(\frac{a}{1-a} \right) \exp(-\mu_{t+1})}, \quad (6.24)$$

$$C_{2t} = \frac{(Y_t - X - G_t)}{1 + \beta \left(\frac{a}{1-a} \right) \exp(-\mu_{t+1})}. \quad (6.25)$$

Inserting (6.24) into the cash-in-advance constraint in (2.6), which holds with equality in equilibrium as long as the real interest rate is positive, produces the closed-form equation for the price level,

$$P_t = \frac{M_t}{(Y_t - X - G_t)} \left[\frac{1 + \beta \left(\frac{a}{1-a} \right) \exp(-\mu_{t+1})}{\beta \left(\frac{a}{1-a} \right) \exp(-\mu_{t+1})} \right], \quad (6.26)$$

while the closed-form solutions for the real and nominal interest rate are found by inserting (6.25) and (6.26) at time t and $t + 1$ into (6.14) and (6.15).

Finally, the solution in (6.25) can be substituted into the Euler condition for labor in (6.13) to solve for optimal labor supply. Doing so, and noting the specification for output in (2.1), defines an implicit function,

$$F(H_t, g_t, \theta_t, \mu_{t+1}, \tau_t) = 0. \quad (6.27)$$

This equation cannot be solved for H_t explicitly, but the implicit function theorem will allow for the construction of an implicit function which defines the explicit function. The defined derivatives can be obtained as long as an implicit function is known to exist under the implicit function theorem.

Proposition 1. *The function $F(H_t, g_t, \theta_t, \mu_{t+1}, \tau_t) = 0$ defines an implicit function $H_t = h(g_t, \theta_t, \mu_{t+1}, \tau_t)$.*

The implicit function theorem states that given $F(H_t, g_t, \theta_t, \mu_{t+1}, \tau_t) = 0$, if (a) the function F has continuous partial derivatives F_H , F_g , F_θ , F_μ , and F_τ and, (b) at a point

$(H_0, g_0, \theta_0, \mu_0, \tau_0)$ satisfying $F(H_t, g_t, \theta_t, \mu_{t+1}, \tau_t) = 0$, F_H is non-zero except when $H = 0$, then there exists a 4-dimensional neighborhood of $(g_0, \theta_0, \mu_0, \tau_0)$, N , in which h is an implicitly defined function of the variables g, θ, μ , and τ in the form of $h(g_t, \theta_t, \mu_{t+1}, \tau_t)$.³⁷

The continuous partial derivatives of (6.27) are

$$F_H : \frac{\alpha Y_t}{H_t} \left[\frac{\gamma}{1 + \beta \left(\frac{a}{1-a} \right) \exp(-\mu_{t+1})} + \frac{(1-\alpha)(1-\alpha\tau_t)}{H_t} \right], \quad (6.28)$$

$$F_g : \frac{-\gamma G_t}{1 + \beta \left(\frac{a}{1-a} \right) \exp(-\mu_{t+1})}, \quad (6.29)$$

$$F_\theta : \frac{-\gamma}{1 + \beta \left(\frac{a}{1-a} \right) \exp(-\mu_{t+1})}, \quad (6.30)$$

$$F_\mu : \frac{\gamma C_{1t}}{1 + \beta \left(\frac{a}{1-a} \right) \exp(-\mu_{t+1})}, \quad (6.31)$$

$$F_\tau : (1-\alpha) \alpha^2 \frac{Y_t}{H_t}. \quad (6.32)$$

Given that $0 < \alpha, \beta < 1$, and γ is defined as a positive constant, F_H is non-zero except when $H = 0$, where F_H becomes undefined. Thus, around any point on the function, except $H = 0$, a neighborhood, N , can be constructed in which $F(H_t, g_t, \theta_t, \mu_{t+1}, \tau_t) = 0$ defines an implicit function $H_t = h(g_t, \theta_t, \mu_{t+1}, \tau_t)$.

Further examination of the labor supply function shows that optimal labor supply will be bounded away from zero and unique over the interval examined. Equation (6.27) acts as the difference function between the left and right-hand sides of equation (6.13). The left-hand side of equation (6.13) is upward sloping in labor supply while the right-hand side is downward sloping in labor supply. The left-hand side contains the term for overall consumption, $(Y_t - X_t - G_t)$ and when calibrated to match the features of the U.S. economy and examined over the interval $[0, 1]$ in labor supply, begins below zero and slowly increases. At low levels of labor supply, total output is less than government spending. As additional labor supply is added, output quickly outpaces government spending. The function is always upward sloping over the interval in question. The term on the right-hand side contains the marginal produce of labor and is downward sloping in labor supply. The calibrated function begins at higher levels with low labor supply since marginal productivity of labor is high and slowly decreases as labor is increased. Consequently, the difference function begins negative at low levels of labor supply (low total consumption relative to high marginal product of labor) and turns positive as labor supply is increased (high total consumption relative to low marginal product of labor). Since the difference function is continuous and maintains a positive slope over the interval in question, the optimal labor supply which equates the two sides and satisfies the Euler condition is strictly greater than zero and is unique over the $[0, 1]$ interval.

In equilibrium, optimal labor supply is a function of government policy and the ex-

³⁷See Sydsaeter (1981, 81)

ogenous shocks to government spending and technology. Furthermore, since an implicit function for labor supply can be constructed, the optimal allocation of consumption and labor decisions by household, as well as the equilibrium wage rate, are all functions of government policy and the exogenous shocks to government spending and technology. In functional form,

$$C_{1t} = c_1(H_t, g_t, \theta_t, \mu_{t+1}, \tau_t), \quad (6.33)$$

$$C_{2t} = c_2(H_t, g_t, \theta_t, \mu_{t+1}, \tau_t), \quad (6.34)$$

$$H_t = h(g_t, \theta_t, \mu_{t+1}, \tau_t), \quad (6.35)$$

$$w_t = \varpi(g_t, \theta_t, \mu_{t+1}, \tau_t). \quad (6.36)$$

The remaining variables are functions of contemporaneous policy, past policy, or expectations over future outcomes,

$$P_t = p(H_t, g_t, \theta_t, \mu_{t+1}, M_t, \tau_t), \quad (6.37)$$

$$R_t^L = r_L(g_t, \theta_t, \mu_{t+1}, \tau_t, E_t [H_{t+1}, g_{t+1}, \theta_{t+1}, \mu_{t+2}, \tau_{t+1}]), \quad (6.38)$$

$$R_t^N = r_N(E_t [\mu_{t+2}]). \quad (6.39)$$

Examination of the household labor first-order condition in (6.6) reveals the standard upward sloping relationship between household labor supply and the real wage. Holding the marginal utility of consumption constant, the partial derivative with respect to the real wage is positive, or $\partial\gamma/\partial w_t > 0$. Since the marginal disutility of supplying additional labor is a constant, this implies that labor supply has increased and results in an upward sloping household labor supply versus the real wage. From the firm's problem, it is easily seen that labor demand, H_t^d , is negatively related to the real wage since

$$\frac{\partial H_t^d}{\partial w_t} = \frac{1}{1-\alpha} \left[\frac{1}{\alpha \exp(\theta_t) K_t^{1-\alpha}} \right]^{\frac{1}{\alpha-1}-1} < 0. \quad (6.40)$$

The above relationship is negative since $\alpha = 0.6$, making the entire expression negative.

References

- Aiyagari, S. Rao and Ellen McGrattan, 1998. "The Optimum Quantity of Debt." *Journal of Monetary Economics* 42: 447-469.
- Aiyagari, S. Rao, Albert Marcet, Thomas J. Sargent, and Juha Seppala, 2002. "Optimal Taxation without State-Contingent Debt." *Journal of Political Economy* 110: 1220-1254.
- Aiyagari, S. Rao, R. Anton Braun, and Zvi Eckstein, 1998. "Transaction Services, Inflation, and Welfare." *Journal of Political Economy* 106: 1274-1301.
- Aizerman, Joshua and Nancy P. Marion, 1993. "Policy Uncertainty, Persistence and Growth." *Review of International Economics* 1: 145-163.
- Alvarez, Fernando, Patrick J. Kehoe, and Pablo Andrés Neumeyer, 2004. "The Time Consistency of Optimal Monetary and Fiscal Policies." *Econometrica* 72: 541-567.
- Atkinson, A. B. 1971. "Capital Taxes, the Redistribution of Wealth and Individual Savings." *Review of Economic Studies* 38: 209-227.
- Atkinson, A. B. and A. Sandmo. 1980. "Welfare Implications of the Taxation of Savings." *Economic Journal* 90: 529-549.
- Bach, G.L. and R. A. Musgrave, 1941. "A Stable Purchasing Power Bond." *American Economic Review* 823-825.
- Barro, Robert J. 1979. "On the Determination of the Public Debt." *Journal of Political Economy* 87: 940-971.
- Barro, Robert J. 1987. "Government Spending, Interest Rates, Prices, and Budget Deficits in the United Kingdom, 1701-1918." *Journal of Monetary Economics* 20: 221-247.
- Barro, Robert J. and David B. Gordon. 1981. "A Positive Theory of Monetary Policy in a Natural Rate Model." *Journal of Political Economy* 91: 589-610.
- Barro, Robert J. and Chaipat Sahasakul, 1983. "Measuring the Average Marginal Tax Rate from the Individual Income Tax." *Journal of Business* 56: 419-452.
- Benveniste, Lawrence, and Jose Scheinkman. 1979. "On the Differentiability of the Value Function in Dynamic Models of Economies." *Econometrica* 47: 727-732.
- Bernanke, Ben S., 1983. "Irreversibility, Uncertainty, and Cyclical Investment." *Quarterly Journal of Economics* 98(1): 85-106.
- Black, Fischer, 1987. *Business Cycles and Equilibrium*. Cambridge: Basil Blackwell.
- Bohn, Henning, 1988. "Why Do We Have Nominal Government Debt?" *Journal of Monetary Economics* 21: 127-140.
- Chamley, Christophe. 1986. "Optimal Taxation of Capital Income in General Equilibrium with Infinite Lives." *Econometrica* 54: 607-622.
- Chari, V.V. and Patrick J. Kehoe, 1990, "Sustainable Plans." *Journal of Political Economy* 98: 783-802.
- Chari, V.V. and Patrick J. Kehoe, 1993, "Sustainable Plans and Debt." *Journal of Economic Theory* 61: 230-261.

- Chari, V.V. and Patrick J. Kehoe. 1999. "Optimal Fiscal and Monetary Policy." In *Handbook of Macroeconomics* Volume 1C, edited by John B. Taylor and Michael Woodford. North-Holland: Elsevier.
- Chari, V.V., Patrick J. Kehoe, and E.C. Prescott, 1989, "Time Consistency and Policy." In Robert J. Barro, editor, *Modern Business Cycle Theory*. Harvard University Press, Cambridge, MA, 265-305.
- Chari, V.V., Lawrence J. Christiano and Patrick J. Kehoe. 1991. "Optimal Fiscal and Monetary Policy: Some Recent Results." *Journal of Money, Credit and Banking*. 23: 519-539.
- Chari, V.V., Lawrence J. Christiano and Patrick J. Kehoe. 1994. "Optimal Fiscal Policy in a Business Cycle Model." *Journal of Political Economy* 102: 617-652.
- Chari, V.V., Lawrence J. Christiano and Patrick J. Kehoe. 1996. "Optimality of the Friedman Rule in Economies with Distorting Taxes." *Journal of Monetary Economics* 37: 203-223.
- Christiano, Lawrence J. and Martin Eichenbaum. 1992. "Current Real Business Cycle Theories and Aggregate Labor-Market Fluctuations." *American Economic Review* 82: 430-450.
- Cooley, Thomas F. and Gary D. Hansen, 1991. "The Welfare Costs of Moderate Inflation." *Journal of Money, Credit, and Banking* 23: 483-503.
- Cooley, Thomas F. and Gary D. Hansen. 1995. "Money and the Business Cycle." In *Frontiers of Business Cycle Research*. Princeton, NJ: Princeton University Press.
- Cooley, Thomas F. and Edward C. Prescott, 1995. "Economic Growth and Business Cycles." In Thomas F. Cooley, editor, *Frontiers of Business Cycle Research*. Princeton, NJ: Princeton University Press.
- Cosimano, Thomas F. and Michael T. Gapen. 2004. "Solving Ramsey Problems with Nonlinear Projection Methods." Forthcoming in *Studies in Nonlinear Dynamics & Econometrics* 9: Spring 2005. Available at <http://www.nd.edu/%7Eetcosiman/workingpapers.html>.
- Diamond, Peter A. 1973. "Taxation and Public Production in a Growth Setting." In *Models of Economic Growth*, edited by James A. Mirrlees and N. H. Stern. New York: Wiley.
- Friedman, Milton. 1969. "The Optimum Quantity of Money." In *The Optimum Quantity of Money and Other Essays*, edited by Milton Friedman. Chicago: Aldine.
- Goldstein, Morris, 2003. "Debt Sustainability, Brazil, and the IMF." *Institute for International Economics Working Paper* WP03-1.
- Hansen, Gary D. 1985. "Indivisible Labor and the Business Cycle." *Journal of Monetary Economics* 16: 309-327.
- Hansen, Gary D. and Randall Wright. 1992. "The Labor Market in Real Business Cycle Theory." *Federal Reserve Bank of Minneapolis Quarterly Review* 16: 2-12.
- Hodrick, Robert J. and Edward C. Prescott. 1997. "Postwar U.S. Business Cycles: An Empirical Investigation." *Journal of Money, Credit and Banking* 29: 1-16.

- Judd, Kenneth L. 1985. "Redistributive Taxation in a Simple Perfect Foresight Model." *Journal of Public Economics* 28: 59-83.
- Judd, Kenneth L.. 1992. "Projection Methods for Solving Aggregate Growth Models." *Journal of Economic Theory* 58: 410-452.
- Judd, Kenneth L. 1998. *Numerical Methods in Economics*. Cambridge, MA: The MIT Press.
- Juster, F. Thomas, and Frank P. Stafford, 1991. "The Allocation of Time: Empirical Findings, Behavioral Models, and Problems of Measurement." *Journal of Economic Literature* 29: 471-522.
- Kim, J. and S. H. Kim. 2003. "Spurious Welfare Reversals in International Business Cycle Models." *Journal of International Economics* 60: 471-500.
- Ljungqvist, Lars and Thomas J. Sargent. 2000. *Recursive Macroeconomic Theory*. Cambridge, MA: The MIT Press.
- Lucas, Robert E., 1987. *Models of Business Cycles*. Oxford: Basil Blackwell.
- Lucas, Robert E. and Nancy L. Stokey. 1983. "Optimal Fiscal and Monetary Policy in an Economy Without Capital." *Journal of Monetary Economics* 12: 59-93.
- Mankiw, N. Gregory, 1987. "The Optimal Collection of Seigniorage: Theory and Evidence." *Journal of Monetary Economics* 20: 327-341.
- McCallum, Bennett T., 1984. "Credibility and Monetary Policy." *NBER Working Paper* No. 1490.
- Musgrave, R.A. 1959. *The Theory of Public Finance*. New York, NY: McGraw-Hill.
- Pestieau, Pierre M. 1974. "Optimal Taxation and Discount Rate for Public Investment in a Growth Setting." *Journal of Public Economics* 3: 217-235.
- Reinhart, Carmen M., Kenneth S. Rogoff and Miguel A. Savastano, 2003. "Debt Intolerance." *Brookings Papers on Economic Activity*, Spring.
- Ramey, Garey and Valerie A. Ramey, 1991. "Technology Commitment and the Cost of Economic Fluctuations." *NBER Working Paper* No. 3755.
- Ramey, Garey and Valerie A. Ramey, 1995. "Cross-Country Evidence on the Link Between Volatility and Growth." *American Economic Review* 85(5): 1138-1151.
- Ramsey, F. P. 1927. "A Contribution to the Theory of Taxation." *Economic Journal* 37: 47-61.
- Rogerson, R. 1988. "Indivisible Labor, Lotteries, and Equilibrium." *Journal of Monetary Economics* 21: 3-16.
- Schmitt-Grohé, Stephanie and Martín Uribe. 2004. "Optimal Fiscal and Monetary Policy Under Sticky Prices." *Journal of Economic Theory* 114: 198-230.
- Stock, James H. and Mark W. Watson. 1999. "Business Cycle Fluctuations in U.S. Macroeconomic Time Series." In *Handbook of Macroeconomics*, Vol. 1A, edited by John B. Taylor and Michael Woodford. Amsterdam, The Netherlands: Elsevier Science.
- Stokey, Nancy L., 1991, "Credible Public Policy." *Journal of Economic Dynamics and Control* 15: 627-656.

Sydsaeter, Knut. 1981. *Topics in Mathematical Analysis for Economists*. New York, NY: Academic Press.

Table 1: Parameter values corresponding to U.S. economy.

Parameter Values								
α	β	a	γ	δ	ρ_θ	σ_θ	ρ_g	σ_g
0.6	0.991	0.44	2.47	0.017	0.95	0.007	0.96	0.021

Table 2: Model simulations: Steady state values.

Variable	<i>U.S. Debt-to-Income</i>					<i>High Debt-to-Income</i>				
	Percent Nominal Debt					Percent Nominal Debt				
	0%	25%	50%	75%	100%	0%	25%	50%	75%	100%
	(in levels)									
Output	1.736	1.736	1.736	1.736	1.736	1.731	1.731	1.731	1.731	1.731
Cash Good	0.485	0.485	0.485	0.485	0.485	0.483	0.483	0.483	0.483	0.483
Credit Good	0.620	0.620	0.620	0.620	0.620	0.617	0.617	0.617	0.617	0.617
Labor	0.309	0.309	0.309	0.309	0.309	0.308	0.308	0.308	0.308	0.308
Multiplier	0.138	0.138	0.138	0.138	0.138	0.144	0.144	0.144	0.143	0.143
	(in percent)									
Inflation Rate	-0.9%	-0.9%	-0.9%	-0.9%	-0.9%	-0.9%	-0.9%	-0.9%	-0.9%	-0.9%
Nominal Interest Rate	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
Real Interest Rate	0.9%	0.9%	0.9%	0.9%	0.9%	0.9%	0.9%	0.9%	0.9%	0.9%
Money Growth Rate	-0.9%	-0.9%	-0.9%	-0.9%	-0.9%	-0.9%	-0.9%	-0.9%	-0.9%	-0.9%
Tax Rate ^{1/}	18.8%	18.8%	18.8%	18.8%	18.8%	19.3%	19.3%	19.3%	19.3%	19.3%
Tax Rate ^{2/}	31.4%	31.4%	31.4%	31.3%	31.3%	32.2%	32.2%	32.2%	32.2%	32.2%

1/ In percent of income.

2/ In percent of labor income.

Table 3: Model simulations: Standard deviations.

Variable	<i>U.S. Debt-to-Income</i>					<i>High Debt-to-Income</i>				
	Percent Nominal Debt					Percent Nominal Debt				
	0%	25%	50%	75%	100%	0%	25%	50%	75%	100%
Output	0.78	0.70	0.64	0.58	0.54	0.79	0.63	0.55	0.50	0.44
Cash Good	1.41	1.25	1.11	1.01	0.92	1.45	1.11	0.91	0.78	0.69
Credit Good	1.36	1.22	1.11	1.01	0.92	1.34	1.09	0.94	0.83	0.69
Labor	0.22	0.37	0.50	0.59	0.68	0.21	0.49	0.67	0.81	0.92
Multiplier	4.98	3.73	2.94	2.46	2.31	5.29	3.09	2.37	2.45	2.95
Price Level	1.41	1.24	1.12	1.01	0.92	1.49	1.15	0.97	0.85	0.69
Inflation	1.05	0.93	0.83	0.76	0.69	1.06	0.84	0.70	0.61	0.52
Nominal Interest Rate ^{1/}	0.07	0.05	0.03	0.02	0.00	0.14	0.09	0.08	0.07	0.00
Real Interest Rate ^{1/}	0.07	0.06	0.05	0.05	0.04	0.06	0.05	0.05	0.04	0.03
Debt	0.07	0.36	0.59	0.77	0.93	0.07	0.32	0.48	0.60	0.70
Money Growth Rate	0.07	0.05	0.03	0.02	0.00	0.15	0.09	0.08	0.08	0.00
Tax Rate ^{2/}	2.66	2.10	1.88	1.85	2.04	2.46	1.65	1.75	2.16	2.80

1/ Gross rate.

2/ In percent of income.

Table 4: Simulated economy with U.S. debt-to-income ratio.

100% Indexed Debt

Variable	Cross-Correlation of Output with:							Cross-Correlation				
	x(-3)	x(-2)	x(-1)	x	x(+1)	x(+2)	x(+3)	Money Growth	Tax Rate	Mult.	Shocks	
											Tech.	Gov.
Output	0.26	0.46	0.71	1.00	0.71	0.46	0.26	-0.37	-0.37	-0.55	1.00	-0.02
Cash Good	0.24	0.42	0.63	0.89	0.63	0.42	0.24	-0.75	-0.75	-0.87	0.89	-0.47
Credit Good	0.24	0.42	0.64	0.90	0.64	0.42	0.24	-0.73	-0.73	-0.85	0.90	-0.44
Labor	-0.26	-0.46	-0.71	-1.00	-0.71	-0.46	-0.26	0.36	0.36	0.54	-1.00	0.01
Multiplier	-0.15	-0.27	-0.40	-0.55	-0.40	-0.26	-0.15	0.96	0.98	1.00	-0.55	0.84
Gov. Spending	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	0.92	0.94	0.84	-0.02	1.00
Price Level	-0.20	-0.38	-0.61	-0.88	-0.64	-0.44	-0.27	0.73	0.73	0.85	-0.88	0.45
Inflation	-0.20	-0.25	-0.30	-0.36	0.32	0.27	0.23	0.34	0.34	0.38	-0.36	0.23
Nom. Int. Rate	-0.11	-0.18	-0.27	-0.37	-0.27	-0.18	-0.11	1.00	0.98	0.96	-0.37	0.92
Real Int. Rate	-0.25	-0.44	-0.66	-0.93	-0.66	-0.44	-0.25	0.68	0.67	0.81	-0.93	0.37
Debt	-0.10	-0.24	-0.42	-0.63	-0.88	-0.63	-0.41	0.47	0.46	0.54	-0.63	0.26
Money Growth	-0.11	-0.18	-0.27	-0.37	-0.27	-0.18	-0.11	1.00	0.98	0.96	-0.37	0.92
Tax Rate	-0.11	-0.18	-0.27	-0.37	-0.27	-0.18	-0.11	0.98	1.00	0.98	-0.37	0.94

50% Indexed Debt, 50% Nominal Debt

Variable	Cross-Correlation of Output with:							Cross-Correlation				
	x(-3)	x(-2)	x(-1)	x	x(+1)	x(+2)	x(+3)	Money Growth	Tax Rate	Mult.	Shocks	
											Tech.	Gov.
Output	0.27	0.47	0.72	1.00	0.72	0.47	0.27	0.38	0.38	-0.03	0.99	0.12
Cash Good	0.22	0.39	0.59	0.83	0.60	0.40	0.23	-0.21	-0.20	-0.58	0.89	-0.45
Credit Good	0.22	0.40	0.60	0.84	0.61	0.41	0.23	-0.18	-0.18	-0.56	0.90	-0.43
Labor	-0.25	-0.44	-0.66	-0.93	-0.67	-0.44	-0.25	0.00	-0.01	0.40	-0.97	0.25
Multiplier	0.00	-0.01	-0.02	-0.03	-0.02	-0.02	-0.02	0.91	0.91	1.00	-0.15	0.99
Gov. Spending	0.04	0.06	0.09	0.12	0.08	0.05	0.01	0.96	0.96	0.99	0.00	1.00
Price Level	-0.24	-0.41	-0.60	-0.83	-0.59	-0.38	-0.21	0.19	0.19	0.56	-0.89	0.44
Inflation	-0.18	-0.23	-0.26	-0.30	0.32	0.27	0.23	0.10	0.10	0.24	-0.33	0.19
Nom. Int. Rate	0.11	0.18	0.27	0.37	0.26	0.17	0.08	1.00	1.00	0.91	0.26	0.96
Real Int. Rate	-0.23	-0.41	-0.63	-0.88	-0.63	-0.42	-0.24	0.10	0.10	0.49	-0.93	0.35
Debt	-0.08	-0.22	-0.39	-0.60	-0.83	-0.60	-0.40	0.15	0.14	0.41	-0.64	0.32
Money Growth	0.11	0.18	0.27	0.38	0.27	0.17	0.08	1.00	1.00	0.91	0.26	0.96
Tax Rate	0.11	0.18	0.27	0.38	0.27	0.17	0.09	1.00	1.00	0.91	0.26	0.96

100% Nominal Debt

Variable	Cross-Correlation of Output with:							Cross-Correlation				
	x(-3)	x(-2)	x(-1)	x	x(+1)	x(+2)	x(+3)	Money Growth	Tax Rate	Mult.	Shocks	
											Tech.	Gov.
Output	0.26	0.47	0.71	1.00	0.71	0.47	0.26	0.86	0.86	0.60	0.97	0.21
Cash Good	0.21	0.37	0.55	0.77	0.55	0.37	0.21	0.35	0.35	-0.04	0.90	-0.45
Credit Good	0.21	0.37	0.55	0.77	0.55	0.37	0.21	0.35	0.35	-0.04	0.90	-0.45
Labor	-0.23	-0.40	-0.61	-0.85	-0.61	-0.40	-0.23	-0.48	-0.48	-0.10	-0.95	0.32
Multiplier	0.15	0.27	0.43	0.60	0.42	0.27	0.15	0.91	0.92	1.00	0.40	0.91
Gov. Spending	0.05	0.09	0.15	0.21	0.15	0.09	0.05	0.67	0.67	0.91	-0.02	1.00
Price Level	-0.21	-0.37	-0.55	-0.77	-0.55	-0.36	-0.21	-0.35	-0.35	0.04	-0.90	0.45
Inflation	-0.17	-0.21	-0.25	-0.29	0.29	0.25	0.21	-0.13	-0.13	0.01	-0.34	0.17
Nom. Int. Rate	0.22	0.40	0.61	0.86	0.61	0.40	0.22	1.00	0.99	0.91	0.72	0.67
Real Int. Rate	-0.22	-0.39	-0.59	-0.82	-0.59	-0.39	-0.22	-0.42	-0.42	-0.04	-0.93	0.38
Debt	-0.08	-0.21	-0.36	-0.55	-0.77	-0.55	-0.36	-0.25	-0.25	0.03	-0.64	0.33
Money Growth	0.22	0.40	0.61	0.86	0.61	0.40	0.22	1.00	0.99	0.91	0.72	0.67
Tax Rate	0.22	0.40	0.61	0.86	0.61	0.40	0.22	0.99	1.00	0.92	0.72	0.67

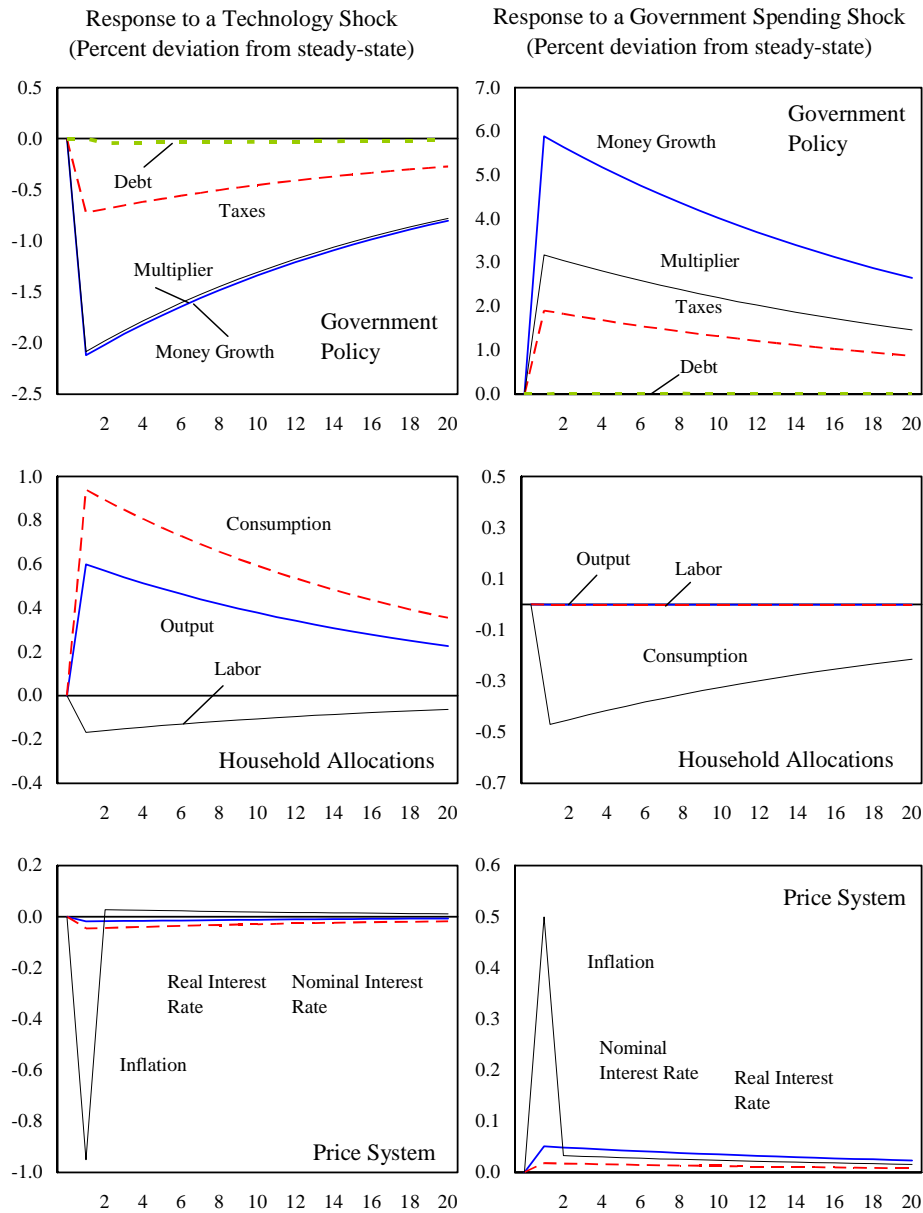
Table 5: Simulated economy with high debt-to-income ratio.

100% Indexed Debt												
Variable	Cross-Correlation of Output with:							Cross-Correlation				
	x(-3)	x(-2)	x(-1)	x	x(+1)	x(+2)	x(+3)	Money Growth	Tax Rate	Mult.	Shocks	
											Tech.	Gov.
Output	0.27	0.47	0.71	1.00	0.71	0.47	0.27	-0.37	-0.37	-0.55	1.00	0.00
Cash Good	0.23	0.41	0.63	0.88	0.63	0.41	0.24	-0.76	-0.76	-0.87	0.88	-0.48
Credit Good	0.24	0.43	0.65	0.91	0.65	0.43	0.25	-0.71	-0.72	-0.84	0.91	-0.41
Labor	-0.27	-0.47	-0.71	-1.00	-0.71	-0.47	-0.27	0.34	0.34	0.52	-1.00	-0.02
Multiplier	-0.14	-0.25	-0.39	-0.55	-0.39	-0.26	-0.15	0.96	0.98	1.00	-0.54	0.84
Gov. Spending	0.00	0.00	0.00	0.00	0.00	-0.01	-0.01	0.91	0.93	0.84	0.00	1.00
Price Level	-0.15	-0.33	-0.56	-0.83	-0.63	-0.45	-0.29	0.69	0.69	0.80	-0.83	0.42
Inflation	-0.20	-0.26	-0.32	-0.38	0.29	0.25	0.21	0.40	0.40	0.44	-0.38	0.28
Nom. Int. Rate	-0.09	-0.17	-0.26	-0.37	-0.26	-0.17	-0.10	1.00	0.98	0.96	-0.36	0.91
Real Int. Rate	-0.25	-0.44	-0.67	-0.94	-0.67	-0.44	-0.25	0.66	0.66	0.79	-0.94	0.34
Debt	-0.10	-0.24	-0.43	-0.65	-0.92	-0.66	-0.43	0.47	0.47	0.55	-0.65	0.25
Money Growth	-0.09	-0.17	-0.26	-0.37	-0.26	-0.18	-0.10	1.00	0.98	0.96	-0.36	0.91
Tax Rate	-0.10	-0.17	-0.26	-0.37	-0.26	-0.18	-0.10	0.98	1.00	0.98	-0.36	0.93

50% Indexed Debt, 50% Nominal Debt												
Variable	Cross-Correlation of Output with:							Cross-Correlation				
	x(-3)	x(-2)	x(-1)	x	x(+1)	x(+2)	x(+3)	Money Growth	Tax Rate	Mult.	Shocks	
											Tech.	Gov.
Output	0.28	0.48	0.71	1.00	0.71	0.48	0.28	0.87	0.87	0.60	0.97	0.25
Cash Good	0.20	0.35	0.53	0.74	0.52	0.35	0.21	0.31	0.31	-0.10	0.88	-0.47
Credit Good	0.22	0.37	0.56	0.79	0.56	0.38	0.22	0.39	0.39	-0.02	0.92	-0.40
Labor	-0.23	-0.39	-0.59	-0.83	-0.59	-0.40	-0.23	-0.46	-0.46	-0.05	-0.94	0.33
Multiplier	0.17	0.29	0.43	0.60	0.43	0.28	0.16	0.90	0.91	1.00	0.38	0.92
Gov. Spending	0.07	0.12	0.18	0.25	0.18	0.12	0.07	0.68	0.69	0.92	0.00	1.00
Price Level	-0.34	-0.45	-0.58	-0.73	-0.45	-0.24	-0.07	-0.34	-0.33	0.06	-0.85	0.41
Inflation	-0.14	-0.16	-0.18	-0.20	0.38	0.30	0.24	-0.03	-0.03	0.11	-0.26	0.23
Nom. Int. Rate	0.24	0.42	0.62	0.87	0.62	0.41	0.24	1.00	0.99	0.90	0.72	0.69
Real Int. Rate	-0.23	-0.40	-0.60	-0.83	-0.59	-0.40	-0.23	-0.46	-0.46	-0.05	-0.94	0.32
Debt	-0.09	-0.20	-0.35	-0.53	-0.74	-0.53	-0.35	-0.23	-0.23	0.07	-0.63	0.33
Money Growth	0.24	0.42	0.62	0.87	0.62	0.41	0.24	1.00	0.99	0.90	0.73	0.68
Tax Rate	0.25	0.42	0.62	0.87	0.62	0.41	0.24	0.99	1.00	0.91	0.73	0.69

100% Nominal Debt												
Variable	Cross-Correlation of Output with:							Cross-Correlation				
	x(-3)	x(-2)	x(-1)	x	x(+1)	x(+2)	x(+3)	Money Growth	Tax Rate	Mult.	Shocks	
											Tech.	Gov.
Output	0.26	0.47	0.71	1.00	0.71	0.47	0.26	0.95	0.99	1.00	0.90	0.43
Cash Good	0.15	0.28	0.44	0.61	0.44	0.29	0.17	0.71	0.74	0.62	0.89	-0.45
Credit Good	0.16	0.28	0.44	0.62	0.44	0.29	0.17	0.71	0.74	0.62	0.90	-0.44
Labor	-0.18	-0.32	-0.49	-0.69	-0.50	-0.33	-0.19	-0.78	-0.80	-0.70	-0.94	0.34
Multiplier	0.26	0.47	0.71	1.00	0.71	0.47	0.26	0.94	0.99	1.00	0.90	0.42
Gov. Spending	0.12	0.20	0.31	0.43	0.30	0.20	0.11	0.27	0.27	0.42	0.00	1.00
Price Level	-0.17	-0.30	-0.45	-0.62	-0.43	-0.28	-0.15	-0.71	-0.74	-0.62	-0.89	0.44
Inflation	-0.13	-0.16	-0.19	-0.23	0.24	0.20	0.17	-0.26	-0.27	-0.23	-0.33	0.17
Nom. Int. Rate	0.25	0.44	0.68	0.95	0.68	0.45	0.25	1.00	0.96	0.95	0.93	0.27
Real Int. Rate	-0.17	-0.31	-0.48	-0.68	-0.48	-0.32	-0.18	-0.76	-0.79	-0.68	-0.92	0.36
Debt	-0.05	-0.15	-0.28	-0.43	-0.61	-0.44	-0.29	-0.50	-0.52	-0.44	-0.64	0.32
Money Growth	0.25	0.44	0.68	0.95	0.68	0.44	0.25	1.00	0.96	0.94	0.93	0.27
Tax Rate	0.26	0.46	0.70	0.99	0.70	0.46	0.26	0.96	1.00	0.99	0.96	0.27

Figure 1: Impulse response functions: Low debt economy with all indexed debt.



Note: Percent deviation of money growth rate from its steady-state value is based on the net money growth rate. Percent deviation of real and nominal interest rates are based on gross interest rates.

Table 6: Directional response to a one-standard deviation shock.

Deviation from Steady-State Value: U.S. Debt-to income ratio									
Variable	<i>100% Indexed Debt</i>					<i>100% Nominal Debt</i>			
	Technology Shock		Govt. Spending Shock			Technology Shock		Govt. Spending Shock	
	Period		Period			Period		Period	
	1	2	1	2	1	2	1	2	
Output	+	+	-	-	+	+	+	+	
Cash Good	+	+	-	-	+	+	-	-	
Credit Good	+	+	-	-	+	+	-	-	
Labor	-	-	-	-	-	-	+	+	
Multiplier	-	-	+	+	+	+	+	+	
Inflation	-	+	+	+	-	+	+	-	
Nominal Interest Rate	-	-	+	+	+	+	+	+	
Real Interest Rate	-	-	+	+	-	-	+	+	
Debt	-	-	+	+	-	-	+	+	
Money Growth Rate	-	-	+	+	+	+	+	+	
Tax Rate	-	-	+	+	+	+	+	+	

Table 7: Hedging value of nominal debt.

Utility Equivalent Over 100% Indexed Debt										
	U.S. Debt-to-Income					High Debt-to-Income				
	Percent Nominal Debt					Percent Nominal Debt				
	0%	25%	50%	75%	100%	0%	25%	50%	75%	100%
(Discounted present value in percent)										
Total Utility	-	0.4	0.6	0.7	0.7	-	0.6	0.7	0.6	0.4
Consumption	-	1.5	2.4	2.9	3.2	-	2.4	3.3	3.6	3.5
Cash Good	-	1.5	2.4	3.0	3.4	-	2.7	3.7	4.2	4.6
Credit Good	-	1.5	2.3	2.7	2.9	-	2.0	2.8	3.0	2.2
Labor	-	-0.4	-0.8	-1.1	-1.3	-	-0.7	-1.2	-1.7	-1.9

Figure 2: Standard deviation of household allocations, government policy, and price system.

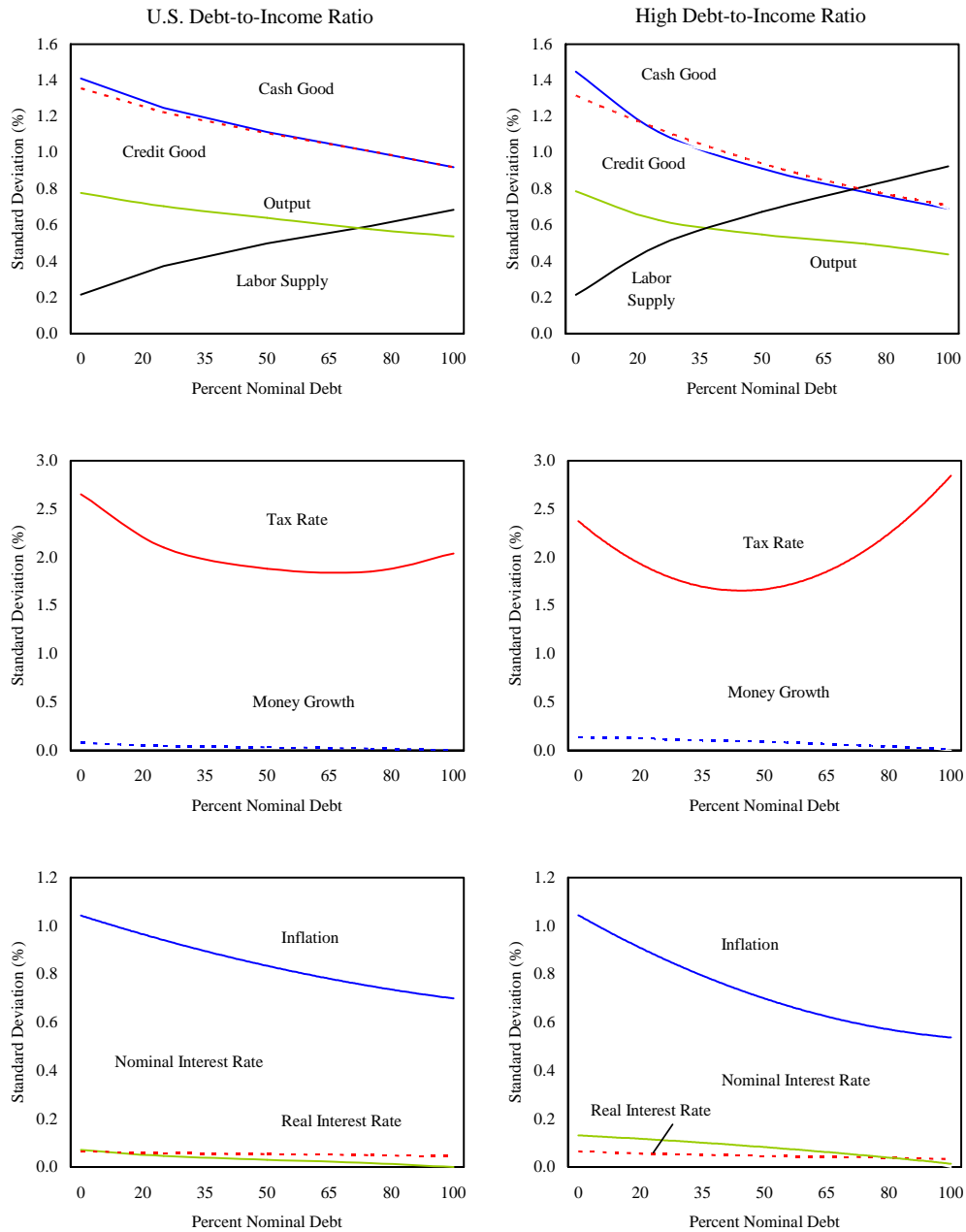
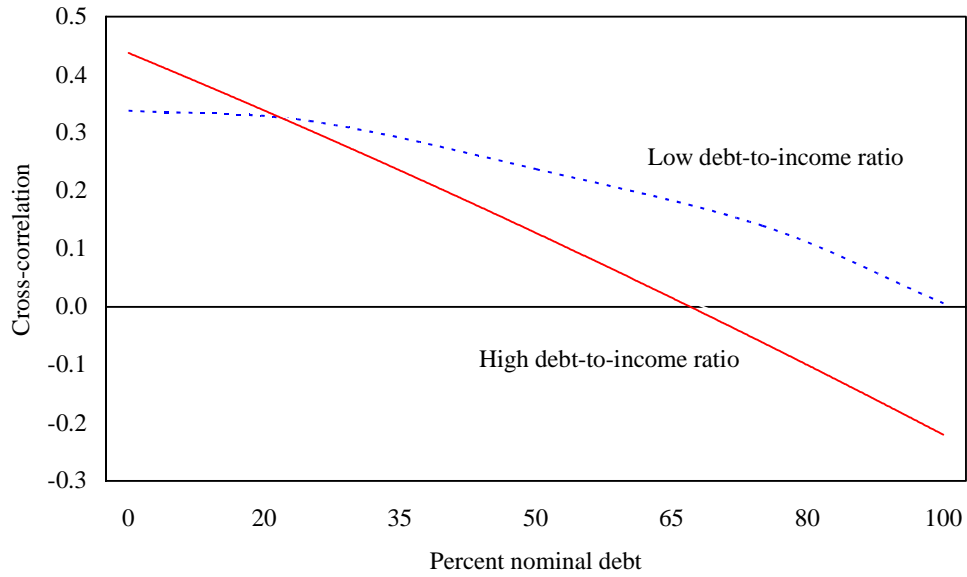


Figure 3: Selected cross-correlations.

Cross-correlation between the multiplier and inflation.



Cross-correlation between government policy and shock to government spending.

